MAT112 Test 3: Extrema, Higher Derivatives, Curve plotting

Name:

Directions: All problems are equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1. A mechanical spot welder on an assembly line moves back and forth as it welds parts onto a car chassis. The position of the tip of the welder is described by the function $s(t)$, where t is time and
$$s(t) = \frac{1}{4}(t-2)^4 - 2(t-2)^2 + 4$$
for $t \in [0, 4]$ (since it takes four seconds for the tip of the welder to make one trip).

1. What is the velocity of the tip as a function of time?

2. What is the acceleration of the tip as a function of time?

3. Where is the tip located at $t = 0$? Where is it at $t = 4$? How far does the tip travel to reach its point of maximum extent?
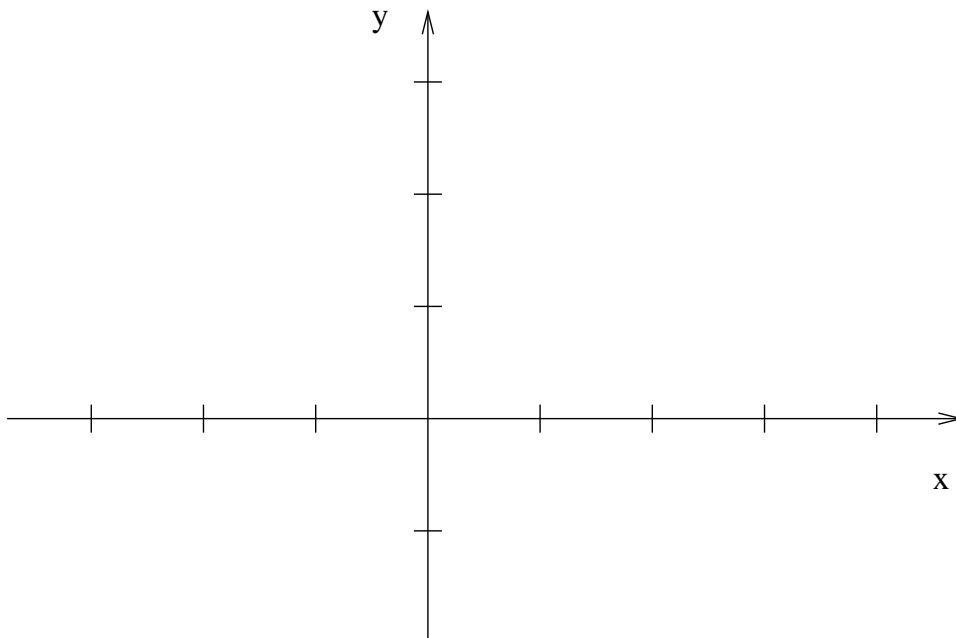
Problem 2. Study the function
$$f(x) = xe^{-x},$$
finding all extrema, points of inflection, asymptotes, and intercepts. Finish with a plot.

y

x

Problem 3. Consider the function
$$f(x) = x^5 - 15x^3$$
defined on the interval $[-4, \infty)$. Find all extrema, and classify them as relative max, relative min, absolute max, or absolute min.

Problem 4. A rectangular fence is being constructed to create an enclosed garden. Three sides are being constructed of cement, but the other (the front) will be constructed of brick for appearance. You'd like the area of the garden to be as large as possible, but you have a limited budget of $10000. The brick costs $20 per linear foot, while cement costs only $10 per linear foot. What are the dimensions of the optimal garden?