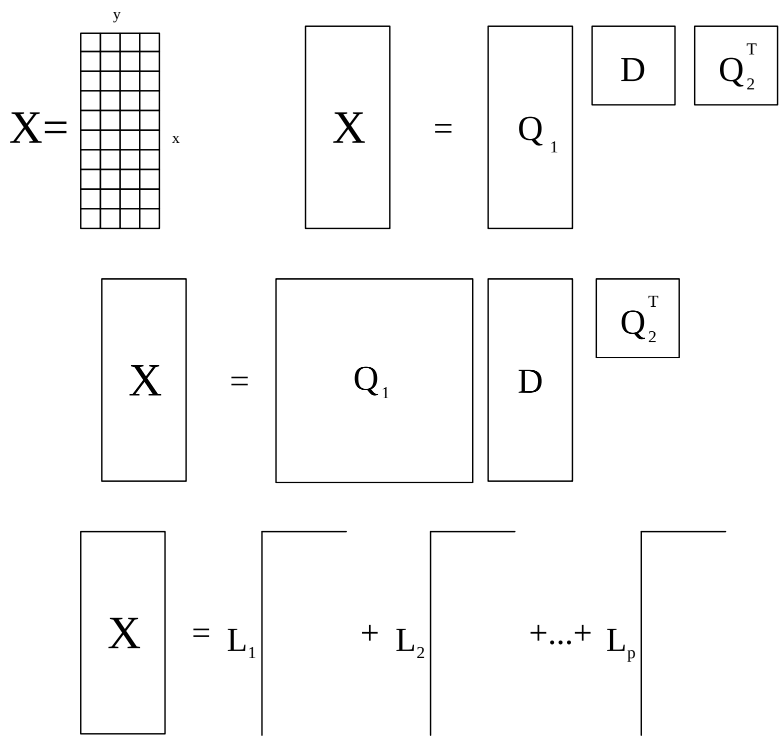

Figure 2.1, Three different operator decompositions of a matrix X.

A second useful “operator-oriented” representation is as a matrix product,

$$X = Q_1 \Lambda Q_2^T,$$

which says that the matrix X can be decoupled as a product of orthogonal matrix Q_1, diagonal matrix Λ, and orthogonal matrix Q_2 (or its transpose, really). This is a good representation from the point of view of the geometrical action of the operator: that it rotates the whole $p-$space, then expands (or contracts) p-space (squashing some dimensions, if any singular values are zero), then rotates back (into $N-$space).

It has been our experience that some problems are more profitably pursued while thinking in terms of the former framework (outer-products), while others are better considered in terms of the latter (products of matrices). The particular generalization of the SVD, alluded to in the title of this chapter, is better imagined