MAT115 Test 2 (Fall 2005): Chapters 3 and 4

Name:

Directions: Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it).

Work exactly 10 of the 12 problems. Write "skip" on the other two. You must skip two! Don't do them all and suppose that I'll drop the two lowest....

Good luck!

Problem 1.

- How many vertices, edges, and faces are possessed by the following Platonic Solids? (You

Table 1:

	vertices	edges	faces
Dodecahedron			
Icosahedron			

may come up and examine the two models at the front of class).

- Note any connection between the two rows of your table, and explain the significance.

- Name all the Platonic solids.

Problem 2. Explain how we know that some infinities are bigger than others.

Problem 3.

- How do we know that the natural numbers $(1, 2, 3, \ldots)$ do not have cardinality greater than all the integers $(\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots)$, without actually constructing a one-to-one correspondence?

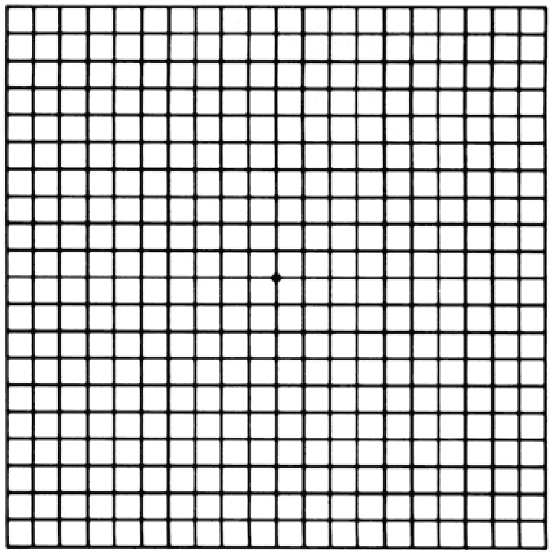
- Demonstrate that the natural numbers are in one-to-one correspondence with all the integers.

Problem 4. Describe a situation in real-life in which a one-to-one correspondence plays a crucial role.

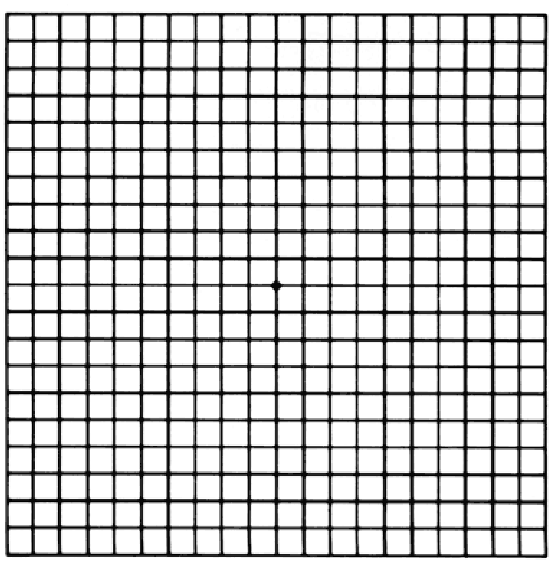
Problem 5. Examine the tiling pattern presented below: assuming that the pattern continues throughout the plane, describe the symmetries this tiling possesses. (Photo of Egyptian pattern

thanks to http://www2.spsu.edu/.)

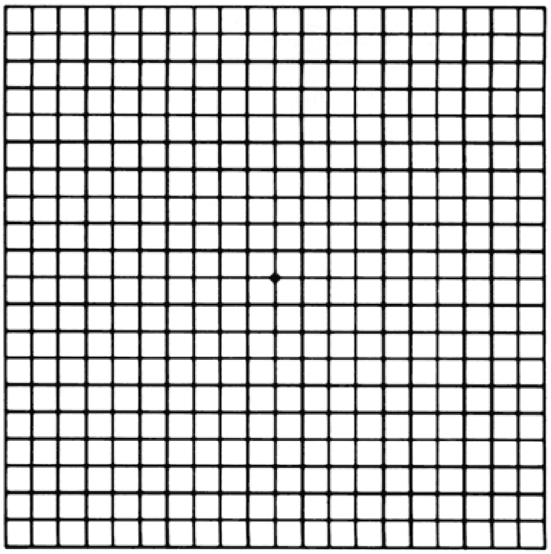
Problem 6. Using the grid cells below, describe the construction of a square-based tiling of the plane that possesses no symmetry whatsoever, and draw it.



Problem 7. Use the grid below to demonstrate the construction of a logarithmic spiral which generates successively better rectangular approximations of a golden rectangle. Use squares of side lengths given by Fibonacci numbers to contruct it: 1x1, 1x1, 2x2, 3x3, 5x5, etc.



Problem 8. Draw the best approximation possible to a golden rectangle using the grid below, longer side running left to right. In what ratio are the side lengths of a true golden rectangle found? Carve out from within your approximation the largest square you can. What can you conclude about the rectangle left over from the operation (include justification!)?



Problem 9. An official NCAA football field is a rectangle 160 feet wide, and 360 feet long (including the end zones). A punt returner receives the ball in the back left corner of his end zone, and runs the ball all the way to the back right corner of his opponent's end zone. What is the minimum distance he ran?

Problem 10. The Pythagoreans were devoted to whole numbers, and believed that the world was fundamentally a place completely describable by whole numbers. What happened to rock their world?

Problem 11. How did the clerk at the Motel ∞ handle an infinite number (natural numbered busses) of natural number-sized busses? Suppose that the motel starts out empty (it just opened for business!), and that all the busses are full. How can each person find their own room (no sharing) in the natural number-sized Motel, with rooms to spare?

Problem 12. It is impossible to tile the plane with regular pentagons, because they "won't fit". Give a mathematical description why.