MAT225 Section Summary: 1.1

1. Definitions

$2x + 4y = -4$
$5x + 7y = 11$

system of two linear equations

- **matrix**
  a rectangular array
- **linear equation**
  an equation of the form

$$\overline{a_1 x_1 + a_2 x_2 + \ldots + a_n x_n = b}$$

  where b and the a_i are generally real or complex numbers, and the x_i are variables.
- **system of linear equations**
  A set of linear equations which are to be true simultaneously
- **solution set** of a linear system
  the set of all solutions of a linear system
- **coefficient matrix**
  matrix of the coefficients of a linear system
- **augmented matrix**
  matrix of the coefficients of a linear system and an added column containing the right hand side of the linear system
- **size of a matrix**
  indicates the number of rows and the number of columns of a matrix (e.g. 3 by 4 - 3 rows, 4 columns)

$A = \begin{bmatrix} 2 & 4 \\ 5 & 7 \end{bmatrix}$

$\underline{b} = \begin{bmatrix} -4 \\ 11 \end{bmatrix}$

$[A | b]$

$\left[\begin{array}{cc|c} 2 & 4 & -4 \\ 5 & 7 & 11 \end{array}\right]_{2\times 3}$

2. Theorems/Formulas

A system of linear equations has either

- no solution, or
- exactly one solution, or
- infinitely many solutions.

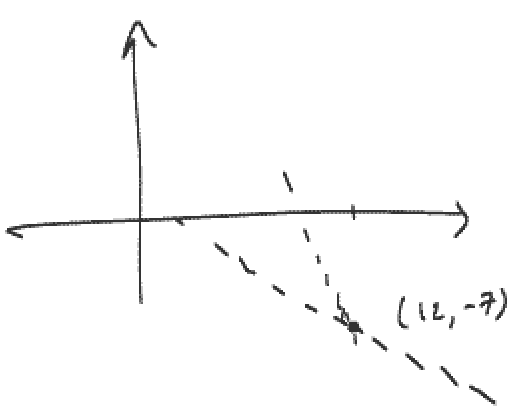
#2, p11:

Q: Given two lines, do they have a unique point of intersection?

A: Yes! (12, -7)

parallel + same $\Rightarrow$ ∞ # of solns

parallel + not : <u>none</u>



3. Properties/Tricks/Hints/Etc.

Fundamental questions about a linear system:

- does a solution exist? (consistency)
- is the solution unique?

Elementary row operations:

$$\begin{bmatrix} 2 & 4 & -4 \\ 5 & 7 & 11 \end{bmatrix}$$

Scaling

$$\begin{bmatrix} 10 & 20 & -20 \\ -10 & -14 & -22 \end{bmatrix}$$

replace

$$\begin{bmatrix} 10 & 20 & -20 \\ \boxed{0} & 6 & -42 \end{bmatrix}$$

zero x locate - solve for y!

- replacement of a row by a sum of the row and multiples of other rows
- interchange of rows
- scaling of a row by a nonzero constant

If the augmented matrices of two linear systems are row equivalent (by the elementary row operations described), then the two systems have the same solution set.

4. Summary

We begin with linear equations, and systems of linear equations. The variable names are essentially irrelevant to the solution set, so matrix notation eliminates the need to even give them names!

Given a system, the idea is to replace the system by one that's easier to solve, yet retains the solutions of the original system. This is done by elementary row operations (replacement, interchange, and scaling). Finally a triangular system is obtained, and the solution can be obtained by back-substitution (if a solution exists!).

$6y = -42 \Leftrightarrow \boxed{y = -7}$

Geometrically, the solution set of a system of linear equations corresponds to the intersection of linear objects embedded in space. There may be no solution, a unique solution, or an infinite number of solutions.

Now back substitute

$10x + 20(-7) = -20$

$10x = 120 \Leftrightarrow \boxed{x = 12}$