MAT225 Section Summary: 1.4

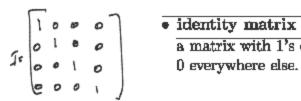
The Matrix Equation Ax = b

1. Definitions

product of matrix A and vector x

If A is an $m \times n$ matrix, with columns a_1, a_2, \ldots, a_n , and if x is in \mathbb{R}^n , then the product of A and x is the linear combination of the columns of A using the corresponding entries in x as weights; that is,

$$A\mathbf{x} = [\mathbf{a}] \mathbf{a}_2 \dots \mathbf{a}_n] \begin{bmatrix} \mathbf{x} \\ \mathbf{x}_n \end{bmatrix} = \mathbf{x}_1 \mathbf{a}_1 + \mathbf{x}_2 \mathbf{a}_2 + \dots + \mathbf{x}_n \mathbf{a}_n$$



identity matrix

a matrix with 1's on the diagonal (top left to bottom right), and

Theorems/Formulas

Theorem Four (p. 43): Let A be an $m \times n$ matrix. Then the following statements are logically equivalent. That is, for a particular A, either they are all true statements or they are all false.

- (a) For each b in \mathbb{R}^m , the equation $A\mathbf{x} = \mathbf{b}$ has a solution.
- (b) Each b in R^m is a linear combination of the columns of A.
- (c) The columns of A span R^m.
- (d) A has a pivot position in every row.

Theorem Five (p. 45): If A is an $m \times n$ matrix, u and v are vectors in \mathbb{R}^n , and c is a scalar, then:

(a)
$$A(\mathbf{u} + \mathbf{v}) = A\mathbf{u} + A\mathbf{v}$$

(b)
$$A(c\mathbf{u}) = c(A\mathbf{u})$$

3. Properties/Tricks/Hints/Etc.

Row-Vector rule for computing Ax:

If the product $A\mathbf{x}$ is defined, then the ith entry in the vector $A\mathbf{x}$ (yes, it's a vector!) is the sum of the products of corresponding entries from row i of A and from the vector \mathbf{x} .

4. Summary

Once again, we yet <u>another</u> representation for a system of linear equations – my god, will it never end? This is the last we'll examine, and probably the most important. Theorem four pulls all these forms together. Spans, pivots, linear combinations, matrix equations collide!

Matrix/vector multiplication is defined. One form that I find particularly useful is the so-called "row-vector rule": a row of the matrix slams into the variable vector \mathbf{x} , to produce a single entry in the b vector.

where

