

§ 2.1 # 22 + 23

Note Title

9/25/2005

22. If the columns of B are l.i.d. then so are the columns of AB

Columns of AB are $A\bar{b}_i$. If $b_i = \alpha_1 \bar{b}_1 + \dots + \alpha_{i-1} \bar{b}_{i-1} + \alpha_{i+1} \bar{b}_{i+1} + \dots + \alpha_n \bar{b}_n$, then $A\bar{b}_i$ is a linear combo of $A\bar{b}_1, \dots, A\bar{b}_{i-1}, A\bar{b}_{i+1}, \dots, A\bar{b}_n$. WLOG - without loss of generality

23. $CA = I_n$ Show that $A\bar{x} = \bar{0}$ has only the trivial soln. Explain why A cannot have more columns than rows.

Suppose $A\bar{x} = \bar{b}$ has non-trivial soln. \bar{y} .

Then $A\bar{y} = \bar{0}$; but also $CA\bar{y} = C\bar{0} = \bar{0}$, so

$$I_n \bar{y} = \bar{0}, \Rightarrow \bar{y} = \bar{0} \text{ contradiction;}$$

Hence \bar{y} is trivial.

If A had more columns than rows, then $A\bar{x} = \bar{b}$ would have a non-trivial solution.