2.6 Muller's method

Note Title 10/3/2005

1d $\quad f(x) = x^4 + 2x^2 - x - 3$

Want $f(1)$, say

$$f(x) = (x-1)\,Q(x) + f(1)$$

$$b_4 = a_4$$
$$b_3 = a_3 + b_4 \cdot 1$$
$$b_2 = a_2 + b_3 \cdot 1$$
$$b_1 = a_1 + b_2 \cdot 1$$
$$b_0 = a_0 + b_1 \cdot 1$$

$$(a_4, a_3, a_2, a_1, a_0)$$
$$= (1, 0, 2, -1, -3)$$

$$(b_4, b_3, b_2, b_1, b_0)$$
$$= (1, 1, 3, 2, -1)$$

$$Q(x) = x^3 + x^2 + 3x + 2$$

$$= (x-1)\,R(x) + Q(1)$$

$$(a_3, a_2, a_1, a_0)$$

$$= (1, 1, 3, 2)$$

$$(b_3, b_2, b_1, b_0)$$

$$= (1, 2, 5, 7) \qquad Q(1) = b_0 = 7$$

Next FPI estimate: $\boxed{g(1) = 1 - \frac{-1}{7} = 8/7}$

Given a set of three points

$(x_0, f(x_0))$
$(x_1, f(x_1))$
$(x_2, f(x_2))$

How do we find the quadratic that passes through the points?

[What must we require?]

We might expand about our most recent point x_2, so

$$f(x) = a(x - x_2)^2 + b(x - x_2) + c$$

$$\begin{bmatrix} (x_2 - x_2)^2 & (x_2 - x_2) & 1 \\ (x_1 - x_2)^2 & (x_1 - x_2) & 1 \\ (x_0 - x_2)^2 & (x_0 - x_2) & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ (x_1 - x_2)^2 & (x_1 - x_2) & 1 \\ (x_0 - x_2)^2 & (x_0 - x_2) & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} f(x_2) \\ f(x_1) \\ f(x_0) \end{bmatrix}$$