Be there!

Thrills!

MAT360 Section Summary: 4.1a

Round-off versus Step-size – Fight of the Century!

1. Summary

Even though we pretend that we are calculating "real" values, we're making errors (truncation, round-off). Consider the case of the forward-difference formula. In this case, we compute with errors:

$$\tilde{f}(x_0) = f(x_0) - e(x_0)$$

and

$$\tilde{f}(x_0 + h) = f(x_0 + h) - e(x_0 + h)$$

where e represents round-off error, and $h > 0$.

Now the absolute error E in our derivative calculation will be made up of two parts:

$$\begin{aligned} &= \left| f'(x_0) - \frac{\tilde{f}(x_0+h) - \tilde{f}(x_0)}{h} \right| \\ E &= \left| f'(x_0) - \frac{f(x_0+h) - e(x_0+h) - f(x_0) + e(x_0)}{h} \right| \\ &= \left| f'(x_0) - f'(x_0) + \frac{h}{2} f''(\xi) + \frac{e(x_0+h) - e(x_0)}{h} \right| \end{aligned}$$

so

$$E = \left| \frac{h}{2} f''(\xi) + \frac{e(x_0 + h) - e(x_0)}{h} \right| \leq \frac{Mh}{2} + \frac{2\epsilon}{h}$$

where $M > 0$ is a bound on the second derivative on the interval of interest, and $\epsilon > 0$ is a bound on the size of a truncation or round-off error.

The upshot: we can't just make h as small as we like, and expect approximations to get better and better: round-off error will ultimately kill us. We need to balance the round-off against small h – and we can even guess what value of h is appropriate, given a particular function (and its second derivative), and the size of round-off errors on your particular machine.

Example: #18