MAT360 Section Summary: 4.4
Composite Integration Schemes

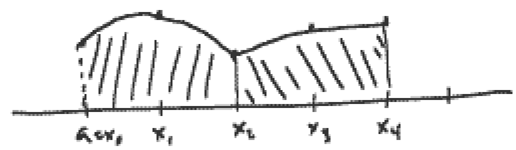
1. Summary

We can continue to generate higher-order Newton-Cotes methods, but the cost is needing to use more and more points, and increasingly complex coefficients schemes. An alternative strategy is to break the interval $[a, b]$ into the elemental patches that we used to define the lower-order methods that we studied in section 4.3, and integrate over the patches and add up the results.

2. Definitions

- **Composite scheme**: divide the interval $[a, b]$ into n patches appropriate to a particular method, apply that method on the patches, then add the results to approximate the integral $\int_a^b f(x)dx$.

3. Theorems/Formulas



Composite Simpson's Rule: dividing the interval $[a, b]$ into $2n+1$ points x_i, $i = 0, ..., n$, with stepsize $h = \frac{b-a}{2n}$ we obtain n panels: on each we use elemental Simpson's, so that we get the following n x $(2n+1)$ matrix that multiplies the vector of points $\overline{f} = (f(x_0), \ldots, f(x_n))$:

$$W = \frac{h}{3}\begin{bmatrix} 1 & 4 & 1 & & & & & & \\ & & 1 & 4 & 1 & & & & \\ & & & & \ddots & \ddots & \ddots & & \\ & & & & & & 1 & 4 & 1 \end{bmatrix}$$

The estimate is then given by

$$I \approx [1\,1\,\cdots\,1]Wf$$

That's an ugly number—
Simpson's composite
rule

The product $[11\cdots 1]W$ is constant, so we can do that product once and for all:

$$I \approx \frac{h}{3}[1\ 4\ 2\ 4\ 2\ \cdots\ 2\ 4\ 2\ 4\ 1]\overline{f}$$

I call this the "1-4-2-4-2-step": just one of the classic dances that arise in numerical analysis.

Composite Trapezoidal Rule: is derived similarly, only using the weights

$$W = \frac{h}{2}\begin{bmatrix} 1 & 1 & & & \\ & 1 & 1 & & \\ & & \ddots & \ddots & \\ & & & 1 & 1 \end{bmatrix}$$

The estimate is then given by

$$I \approx \frac{h}{2}[122\cdots 21]\overline{f}$$

(not nearly such an interesting dance...).

Theorem 4.5: Let $f \in C^2[a,b]$, $h = \frac{b-a}{n}$, and $x_i = a + ih$. Then $\exists \mu \in (a,b)$ for which

$$\int_a^b f(x)dx = \frac{h}{2}[1\ 2\ 2\ \cdots\ 2\ 1]\overline{f} - \frac{b-a}{12}h^2 f''(\mu)$$

Composite Midpoint rule: Theorem 4.6: Let $f \in C^2[a,b]$, n be even, $h = \frac{b-a}{n+2}$, and $x_i = a + (i+1)h$ for $i = -1, \ldots, n+1$. Then $\exists \mu \in (a,b)$ for which

$$\int_a^b f(x)dx = 2h[0\ 1\ 1\ \cdots\ 1\ 0]\overline{f} + \frac{b-a}{6}h^2 f''(\mu)$$

Example: #12, p. 204