The derivation of the error terms is not too very complicated: for example, for Simpson's rule we make an "elemental error" of the form

$$-\frac{h^5 f^{(4)}(\xi_i)}{90}$$

on each panel. Hence the total error for the interval is

$$E = -\sum_{i=1}^{n/2} \frac{h^5 f^{(4)}(\xi_i)}{90}$$

n/2 panels

which can be written as

$$\boxed{E = -\frac{(b-a)h^4 f^{(4)}(\mu)}{180}}$$

by taking μ such that $f^{(4)}(\mu)$ is the mean value of the error terms:

$$f^{(4)}(\mu) = \frac{1}{n/2}\sum_{i=1}^{n/2} f^{(4)}(\xi_i) = \frac{2h}{b-a}\sum_{i=1}^{n/2} f^{(4)}(\xi_i)$$

Now, to look at this error term in a slightly different way,

$$\sum_{i=1}^{n/2} 2h f^{(4)}(\xi_i)$$

is a Riemann sum for the integral

$$\int_a^b f^{(4)}(x)dx$$

so that

$$\int_a^b f^{(4)}(x)dx = f^{(3)}(b) - f^{(3)}(a)$$

so that

$$E \approx -\frac{h^4}{180}(f^{(3)}(b) - f^{(3)}(a))$$

(Compare to Beth's clever approach on #21 of the section 3.4 homework.) Similar tricks works for the other elemental rules.

Example: #16, p. 205