

ϵ - δ defn of a limit

Note Title

8/24/2005

ϵ - δ definition of a limit (1.1) :

Show that

$$\lim_{x \rightarrow 0} x^2 = 0$$

Given $\epsilon > 0$ ("the challenge" - think of a small distance between $f(x)$ & l)

we need to find $\delta > 0$ / (such that)

$$|x - x_0| < \delta \quad \Rightarrow \quad |f(x) - l| < \epsilon .$$

$$\left(\begin{array}{l} x_0 = 0 \\ l = 0 \end{array} \text{ in our case} \right)$$

$$\text{Assume } |f(x)| < \varepsilon \Leftrightarrow |x^2| < \varepsilon$$

$$\Leftrightarrow |x|^2 < \varepsilon$$

$$\Leftrightarrow |x| < \sqrt{\varepsilon}$$

$$\Leftrightarrow |x - 0| < \sqrt{\varepsilon}$$

So take $\delta = \sqrt{\varepsilon}$; then

$$|x - 0| < \delta (= \sqrt{\varepsilon}) \Rightarrow |f(x) - 0| < \varepsilon.$$

We've met
the
challenge!