MAT360 Test 1: Chapters 1 and 2.1-2.4

Name:

Directions: Show your work: answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). When in doubt, draw a graph!
Indicate clearly your answer to each problem (e.g., put a box around it).
Good luck!

Problem 1 (20 pts total).

1. (15pts) Use Newton's method to find solutions accurate to within 10^{-5} to

$$x^2 - 2xe^{-x} + e^{-2x} = 0$$

starting from an initial value of 1. Assume that "accuracy" here means $|p_n - p|$, where p is the true root.

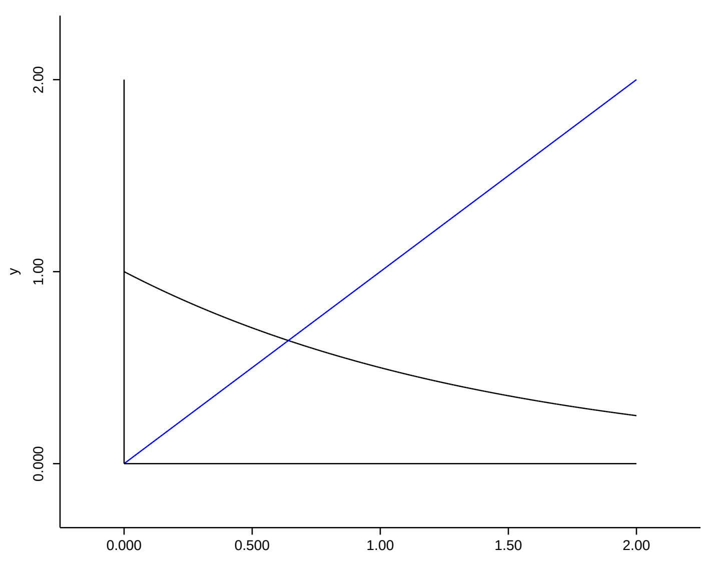
2. (5pts) How do you know that your root is accurate to 10^{-5}?

Problem 2 (20 pts total). Suppose that you're Newton's foolish sibling, and rather than find the optimal fixed point iteration function at root p, with $g'(p) = 0$, you want the sub-optimal (but still pretty good) iteration function with $g'(p) = 1/2$.

1. (10 pts) Start with
$$g(x) = x - \phi(x)f(x)$$
and determine the simplist function $\phi(x)$ that will solve this problem (assume that the root p of f is also simple!).

2. (5 pts) How do we know that the solution of $2^{-x} = x$ is unique?

3. (5 pts) Use this "foolish fixed point function" to find the root of $f(x) = 2^{-x} - x$. Draw the corresponding cobweb diagram.



Problem 3 (20 pts total). Consider the function

$$f(x) = \frac{x}{x^2 + 10^{-6}}$$

whose root is to be determined by bisection.

1. (5 pts) What is <u>the minimum number of iterations</u> to find an approximate root p^* accurate to within 10^{-2} (absolute error in x) for the function on the interval [-1.5,3].

2. (5 pts) Determine the approximate root.

3. (5 pts) Compute f at your approximate root p^*. How are you doing? What's happening to the function values of your iterates? Would you be content to stop at 10^{-2}?

4. (5 pts) Find a pair of distinct initial values (other than the root itself) that would allow the secant method to converge on the root.

Problem 4 (20 pts total).

1. (15 pts) Use three digit chopping and rounding to compute the approximation

$$e \approx \sum_{n=0}^{5} \frac{1}{n!}$$

to determine the most accurate answer possible.

Table 1:

n	chopping	rounding
0		
1		
2		
3		
4		
5		

2. (5 pts) Compare absolute and relative errors of both methods.

Table 2:

method	absolute error	relative error
rounding		
chopping		

Problem 5 (20 pts total). The MacLaurin series expansion of $\sin(x)$ is

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

Use Taylor's Theorem to determine the number of terms that must be kept to ensure that the error in the approximation of $\sin(x)$ by the Taylor series polynomial at every point x on the interval $[0, \pi/2]$ is less than 10^{-6}.

Extra Credit (4pts). Explain the unusual succession of iterates given by attempting to find a root of $f(x) = e^x$ using Newton's method with initial guess 0.