MAT360 Test 2: Sections 2.6, 3.1-3.4, and 4.1-2

Name:

Directions: **Show your work**: answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). When in doubt, draw a graph. If you have a question, ask! Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1 (20 pts total). Consider the problem of fitting a cubic spline through three points, $(x_0, y_0) = (-1, 1)$, $(x_1, y_1) = (0, 0)$, and $(x_2, y_2) = (1, 1)$. Suppose that the points derive from a twice-differentiable even function: obviously we would want our spline to be even, too.

1. (8 pts) Write the two cubics as $s_0(x) = a_0 + b_0x + c_0x^2 + d_0x^3$ and $s_1(x) = a_1 + b_1x + c_1x^2 + d_1x^3$. What additional relationships can you deduce between the coefficients of the two cubics in order for them to satisfy the symmetry condition?

2. (6 pts) Find the natural spline satisfying these constraints. Verify that it satisfies all necessary constraints for a natural spline.

3. (6 pts) Find the clamped spline satisfying these constraints and the derivative conditions $f'(-1) = -2$ and $f'(1) = 2$ (you can do this one by inspection!).

Problem 2 (20 pts total). Consider the function f defined by

$$f(x) = x\sin(x)$$

1. (5 pts) Use Muller's method with $x_0 = 3, x_1 = 2, x_2 = 1$ to find the next iterate in the sequence of approximations to a root.

2. (5 pts) Use the Lagrange form of the interpolating polynomial to write the polynomial that Muller requires to find the iterate x_3.

3. (5 pts) What will happen if you reverse the order of the initial iterates? Why?

4. Use a divided-difference table and forward differences to generate the Newton interpolating polynomial of degree 2 to the three data points. Show your work!

1			
2			
3			

Problem 3 (20 pts total).

- (5 pts) Use forward and backward differences to approximate the derivative of the function $x\sin(x)$ at $x_0 = 0$, with $h = .4$.

- (5 pts) How well does the centered difference scheme do in this case? Why?

- (5 pts) Write the Taylor series expansion about $x_0 = 0$ for f, and use it to deduce the **form** of the error formula for the forward difference approximation to the derivative (that is, what orders of h will factor in?). Assume that $f'(x_0)$ is unknown (and to be estimated), but use the known values of higher derivatives in the expansion.

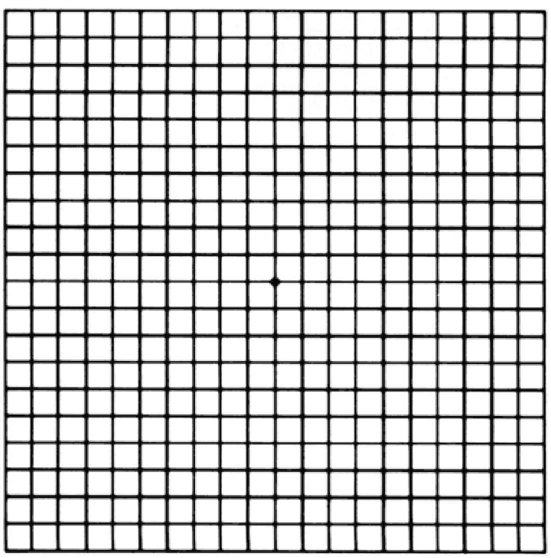
- (5 pts) Use Richardson's Extrapolation on the forward difference formula with h, $h/2$, and $h/4$ to generate $N_3(h)$ for the derivative at $x_0 = 0$. What order error does N_3 achieve?

Problem 4 (20 pts total). Consider the function f defined by

$$f(x) = 3xe^{x} - e^{2x}$$

1. (10 pts) Construct the Hermite interpolating cubic using $x_0 = 1$ and $x_1 = 1.05$.

2. (5 pts) Carefully draw the difference function $H(x) - f(x)$.



3. (5 pts) Approximate $f(1.03)$, and compare the actual error to the error bound (which is the "natural" variant of the Taylor error).