Note Title 1/13/2005

1b n is triangular $\Longleftrightarrow$

$8n+1$ is a perfect square.

$\Rightarrow$: Let n be triangular. Then

$$n = \frac{k(k+1)}{2} \quad \text{for some } k \in \mathbb{N}$$

$$8n+1 = 8\left(\frac{k(k+1)}{2}\right) + 1$$

$$= 4k(k+1) + 1$$
$$= 4k^2 + 4k + 1$$
$$= (2k+1)^2 \quad \checkmark$$

$\Leftarrow$ Let $8n+1$ be a perfect square:

$$\underline{8n+1} = p^2$$

$8n+1$ is odd, so p must be odd: $\quad p = 2k+1$

So $$8n + \cancel{1} = (2k+1)^2$$
$$= 4k^2 + 4k + \cancel{1}$$
$$n = \frac{k(k+1)}{2}$$ ✓

QED

1d If n is triangular, then so are

$9n+1$

$25n+3$

$49n+6$

$81n+10$?

#3a p19

The square of any integer is either of the form $3k$ or $3k+1$.

Any integer can be written in one of three ways: $3q$, $3q+1$, $3q+2$

Check cases!

Let $n \in \mathbb{Z}$ be of the form

1) $n = 3q$

Then $n^2 = (3q)^2 = 3 \cdot (3q^2)$

$(k = 3q^2)$ ✓

2) $n = 3q + 1$

Then $n^2 = (3q+1)^2 = 9q^2 + 6q + 1$

$= 3[3q^2 + 2q] + 1$ ✓

3) $n = 3q + 2$

Then

$$n^2 = (3q+2)^2 = 9q^2 + 12q + 4$$
$$= 3[3q^2 + 4q + 1] + 1$$

#4 Prove that $3a^2 - 1$ is never a perfect square.

$$3a^2 - 1 = 3a^2 - 3 + 2$$
$$= 3[a^2 - 1] + 2$$

invoke #3(a), p 19.

#10 p 20 $n \geq 1$. Demonstrate that $n(7n^2+5)$ is of the form $6k$.

Check cases

$6q$, $6q+1$, $6q+2$, $6q+3$, $6q+4$, $6q+5$