Number Theory Section Summary: 1.1-3

Testing...

Some Preliminary Considerations

"The theory of numbers is concerned, at least in its elementary aspects, with properties of the integers and more particularly with the positive integers 1, 2, 3, ...(also known as the *natural numbers*)...." p. 1.

1. Definitions

Well-Ordering Principle: Every nonempty set S of non-negative integers contains a least element; that is, there is some integer a in S such that $a \leq b$ for every b belonging to S.

2. Theorems

Archimedean property: If a and b are any positive integers, then there exists a positive integer n such that $na \geq b$.

(Proof is by **contradiction**).

First Principle of Finite Induction: Let S be a set of positive integers with the properties that

(a) The integer 1 belongs to S, and

(b) whenever the integer k is in S, then the next integer $k+1$ is also in S.

Then S is the set of all positive integers.

Second Principle of Finite Induction: Let S be a set of positive integers with the properties that

(a) The integer 1 belongs to S, and

(b) whenever the integers $1, \ldots, k$ are in S, then the next integer $k+1$ is also in S.

Then S is the set of all positive integers.

Binomial Theorem:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

$a = 1 = b$

$$2^n = \sum_{n=0}^{n} \binom{n}{k}$$

where

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

3. Properties/Tricks/Hints/Etc. **Pascal's rule**:

$$\binom{n}{k} + \binom{n}{k-1} = \binom{n+1}{k}$$

(this is the source of **Pascal's triangle**).

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

4. Summary

Chapter 1 is preliminary, as the title says. We assume that you've seen this stuff before (except for the history of number theory, which I hope that you'll find interesting!).

The story concerning the drowned disciple of Pythagoras is often told of another student, who may have revealed the existence of irrational numbers (in particular, $\sqrt{2}$). Irrational numbers were not welcomed into polite Pythagorean society...!