MAT310 Test 2: Chapters 4-5

Name:

Directions: Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it).
Note: you may of course use your calculator to check your answers, but the use of the calculator without analysis will not result in many points. These problems could be done before calculators existed! **Good luck!**

Problem 1 (10 pts) Prove (Theorem 4.1): For arbitrary integers a and b, $a \equiv b(\text{mod n})$ if and only if a and b leave the same nonnegative remainder when divided by n.

Problem 2 (10 pts). Show that $18! \equiv -1 \pmod{437}$.

Problem 3 (10 pts). Verify that if $a \equiv b (\text{mod } n_1)$ and $a \equiv b (\text{mod } n_2)$, then $a \equiv b (\text{mod n})$, where $n = \text{lcm}(n_1, n_2)$. Hence, whenever n_1 and n_2 are relatively prime, $a \equiv b (\text{mod } n_1 n_2)$. [**Note**: if you can't do it for n_1 and n_2 in general, can you do it (for 8 pts) for distinct primes p and q?]

Problem 4 (10 pts). Find the last two digits of the number 9^{9^9}. [*Hint*: show that $9^9 \equiv 9 (\text{mod } 10)$. Re-express this fact, using the division algorithm; then use the fact that $9^9 \equiv 89 (\text{mod } 100)$ to complete the picture.]

Problem 5 (10 pts). Find an integer having the remainders 1, 2, 5, 5 when divided by 2, 3, 6, 11, respectively.

Problem 6 (10 pts).

1. Demonstrate that $a^9 \equiv a (\text{mod } 30)$.

2. Calculate (that is, find the least residue of) 5^{23} (mod 127).

Extra Credit (3 pts). What can you tell me about Pierre de Fermat, or the history of Wilson's Lemma? [Choose **one** – not both.]