Highlights, Section 1.2: Propositional Logic

- **Propositional wff**: represent some sort of argument, to be tested, or proven, by **propositional logic**.
- **valid arguments**, e.g.

  $$P_1 \text{ and } P_2 \text{ and } \ldots \text{ and } P_n \text{ -> } Q$$

  have **hypotheses** (we suppose that the P_i are true), and a **conclusion** (Q). To be *valid*, this argument must be a tautology (always true). To be an *argument*, Q must not be identically true (i.e. a fact, in which case the hypotheses would be irrelevant!).

- **Proof Sequence**: a sequence of wffs in which every wff is a hypothesis or the result of applying the formal system's derivation rules (truth-preserving rules) in sequence.

  Objective: to reach the conclusion Q from the hypotheses $P_1, P_2, \ldots, P_n$.

- Types of derivation rules:
  - **Equivalence rules** (see Table 1.12, p. 23):

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    we can substitute equivalent wffs in a proof sequence. One way of showing that two wffs are equivalent is via their truth tables.

    - commutative
    - associative
    - De Morgan's laws
    - implication
    - double negation

    Implication seems somewhat unusual, but it is suggested by Exercise 6a, section 1.1. You're asked to prove it in Practice 9, p. 23. That is, prove that

    $$(P \text{ -> } Q) \text{ <-> } (P' \text{ or } Q)$$

    is a tautology. How would you do it?

  - **Inference rules**: from given hypotheses, we can deduce certain conclusions (see Table 1.13, p. 24)
    - modus ponens:
      If Q follows from P, and P is true, then so is Q.
    - modus tollens:
      If Q follows from P, and Q is false, then so is P.
    - conjunction:
      If Q is true, and P is true, then they're both true together.
    - simplification:
      If both Q and P are true, then they're each true separately.
    - addition:
      If P is true, then either P or Q is true.

    Practice 10, p. 24. Also give step 4!

1.	$(A \wedge B') \to C$	hyp
2.	C'	hyp
3.	$(A \wedge B')'$	1, 2 mt
4.	$A' \vee B'$	3 de Morgan

#27 $P \wedge P' \to Q$

1.	P	hyp
2.	P'	hyp
3.	$P \vee Q$	1, add
4.	$P' \vee Q$	2, add
5.	$P \to Q$	4, imp
6.	Q	1, 5 mp

For a more elaborate example, let's look at #27, p. 32, which shows that one can prove anything if one introduces a contradiction (e.g. the mensa quiz). Also called an **inconsistency**.

- The difference between equivalence rules and inference rules is that equivalence rules are bi-directional (work both ways), whereas some inference rules are uni-directional (work in only one direction - this is what inference is all about: from this we can infer that, but we cannot necessarily infer this from that!).

  Notice that in the table 1.14 (p. 31) some rules appear twice: two uni-directionals can make a bi-directional!

  Note for your homework: you are not allowed to invoke the rule that you are trying to prove! Notice that the entries in this table are followed by exercise numbers: it is in those exercises that the results are obtained!

- **Deduction method**: if we seek to prove an implication, we can simply add the hypothesis of this conclusion implication to the hypothesis of the argument, and prove the conclusion of the remaining implication:

  $$P_1 \text{ and } P_2 \text{ and } \ldots \text{ and } P_n \text{ -> } (R \text{ -> } S)$$

  can be replaced by

  $$P_1 \text{ and } P_2 \text{ and } \ldots \text{ and } P_n \text{ and } R \text{ -> } S$$

  If you're interested in seeing why this rule works, you might try exercise 45, p. 33, but think of it this way: we're interested in assuming that all the P_i are true, and see if we can deduce the implication R-> S. If R is false, then the implication is true. The only potentially problematic case is where R is true, and S is false. Then what we want to know is: given that

  $$P_1 \text{ and } P_2 \text{ and } \ldots \text{ and } P_n \text{ and } R$$

  are true, is S true?

  Exercise #32, p. 32

- **Hypothetical syllogism**:

  if P -> Q and Q -> R, then P -> R.

  (and see a whole long list of rules in Table 1.14). This rule might be referred to as **transitivity**.

A new rule is created each time we prove an argument; but we don't want to create so many rules that we keel over under their weight! Keep just a few rules in view, and learn how to use them well....

- Our goal may well be to turn a "real argument" into a symbolic one. This allows us to test whether the argument is sound (that is, that the conclusion follows from the hypotheses).

  Exercise #39, p. 32.

- The propositional logic system is complete and correct:
  - **complete**: every valid argument is provable.
  - **correct**: **only** a valid argument is provable.

  The derivation rules are truth-preserving, so correctness is pretty clear; completeness is not! How can we tell if we can prove every valid argument?!

#32 $(A' \to B') \wedge (A \to C) \to (B \to C)$

1. $A' \to B'$ hyp
2. $A \to C$ hyp
3. B hyp (deduction method)
4. $B \to A$ 1, cont
5. $B \to C$ 4, 2 hs
6. C 3, 5 mp

QED

#39 J – Jane is more popular
E – Jane will be elected
C – Craig will resign

$(J \to E) \wedge (J \to C) \longrightarrow (J \to (E \wedge C))$

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