Problem 1. Consider the following two cases of purported Boolean algebras. Are they, or are they not? Explain!

a. B is the set of four elements $\{0, 1, a, b\}$, with b the inverse of itself.

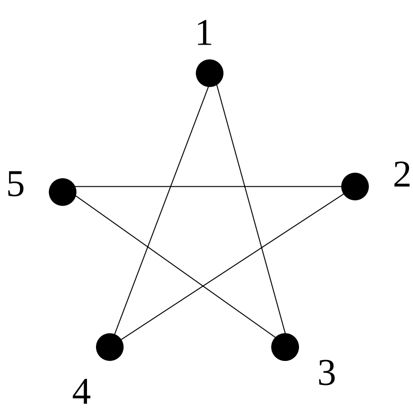
b. Consider the set $B = \{T, F\}$, the binary operation $\vee$ and $\wedge$, and the unary operation $'$ given by

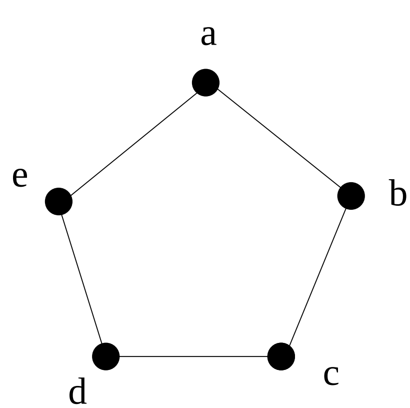
$\vee$	T	F
T	T	T
F	T	F

$\wedge$	T	F
T	T	F
F	F	F

$'$	
T	F
F	T

Problem 2. Determine whether the two graphs below are isomorphic. If so, prove it; if not, explain why not.





Problem 3. Consider the following subsets of the set of all animals:

- A - set of all mammals
- B - set of all reptiles
- C - set of all vertibrates
- D - set of all males

Using set operations, describe each of the following sets in terms of A, B, C, and D:

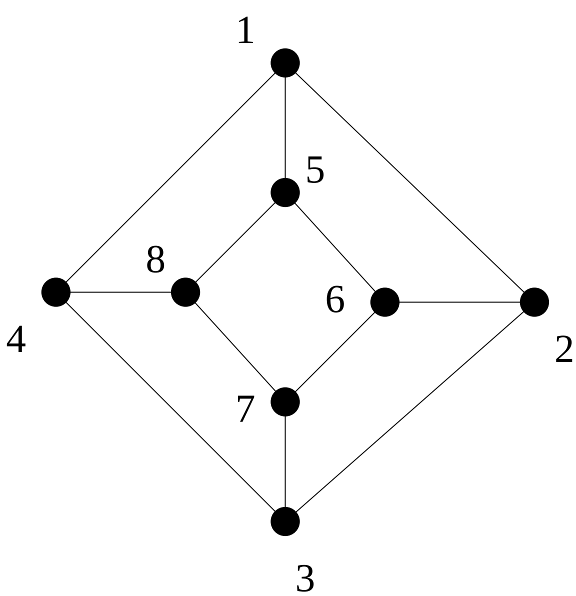
1. set of all female mammals

2. set of all female animals which are not reptiles

3. set of all vertibrates which are not mammals

4. set of all animals which are female or vertibrates

Problem 4. Consider the following graph:



Determine whether

1. An Euler path exists, and

2. Whether a Hamiltonian circuit exists.

Problem 5. The data $\{9, 12, 10, 5, 8, 2, 14\}$ is to be entered into a binary search tree.

1. Enter the data in the order shown, and draw the binary search tree.

2. Find another order for the data entry into a binary search tree which results in a tree of maximal depth.

Problem 6. You are to consider the graph represented by the following adjacency matrix:

$$\begin{bmatrix} 0 & 3 & 5 & \infty & 8 & 1 & \infty & \infty \\ 3 & 0 & 2 & \infty & \infty & \infty & 1 & \infty \\ 5 & 2 & 0 & 1 & \infty & \infty & \infty & 2 \\ \infty & \infty & 1 & 0 & 4 & \infty & \infty & \infty \\ 8 & \infty & \infty & 4 & 0 & 6 & \infty & 1 \\ 1 & \infty & \infty & \infty & 6 & 0 & 5 & \infty \\ \infty & 1 & \infty & \infty & \infty & 5 & 0 & 1 \\ \infty & \infty & 2 & \infty & 1 & \infty & 1 & 0 \end{bmatrix}$$

and suppose that we seek the shortest paths from node 3.

1. Use Bellman-Ford (AnotherShortestPath) to find the initial values of d and s,

	1	2	3	4	5	6	7	8
d								
s								

2. and then their values after the first iteration:

	1	2	3	4	5	6	7	8
d								
s								

Problem 7.

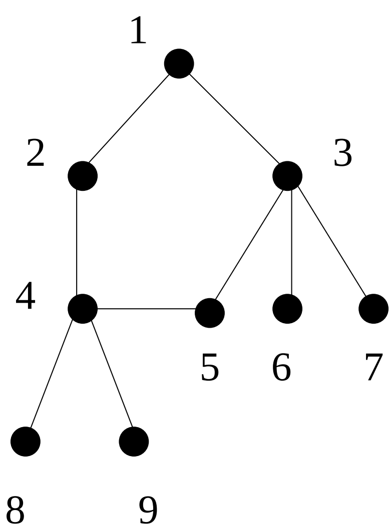
- Draw the graph corresponding to the adjacency matrix of Problem 6.

- Which nodes would be settled after two applications of Dijkstra's algorithm, if we were to start from node 1?

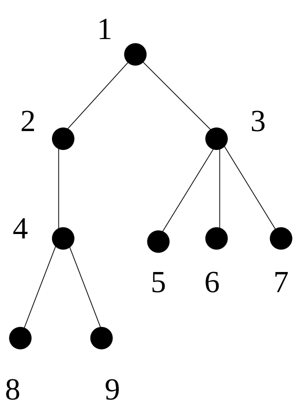
Problem 8. True or False?

1. ( ) A binary tree of depth d has at most 2^d leaves.
2. ( ) Euler's Formula for simple, connected planer graphs means that the addition of a couple of nodes to a graph must result in the increase in the number of edges.
3. ( ) A tree may be cyclic, as we see, for example, in the case of symbolic links in the UNIX file system tree.
4. ( ) A minimal spanning tree is unique.

Problem 9. Carry out both a depth-first and breadth-first traversal of the graph in the following figure, in each case starting from node 4:



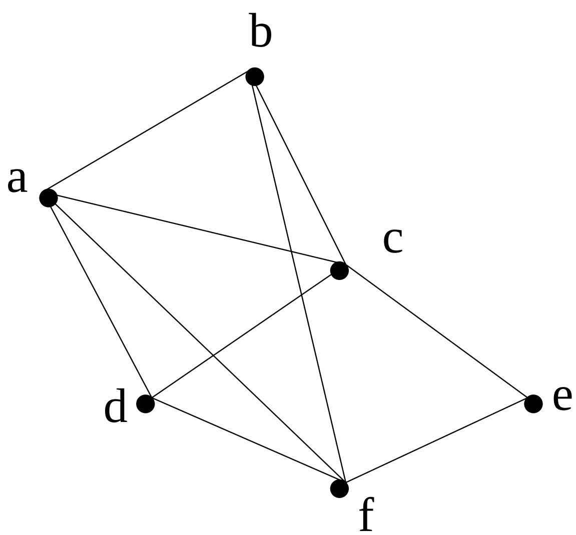
Problem 10. For the tree in the following figure, write the inorder and the postorder traversals of the tree:



Problem 1. Consider the list (f,o,e,n,u,c,r,t,g,m,p).

1. Create a binary search tree from the list (entered in list order).
2. Do a pre-order traversal of the tree.
3. Do an in-order traversal of the tree.
4. Do a post-order traversal of the tree.

Problem 2. Consider the graph below:



1. Produce an adjacency matrix which represents the graph.
2. Determine whether an Euler path exists. If so, indicate how to trace it.
3. Does a Hamiltonian circuit exists using the nodes a-c-d in that order? If so, indicate how to trace it.

Problem 3. Given the following truth function:

x_1	x_2	x_3	$f(x_1, x_2, x_3)$
1	1	1	1
1	1	0	1
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

1. Find the canonical sum of products representation for this truth table (3 pts).
2. Use Boolean algebra to reduce the canonical representation (4 pts).

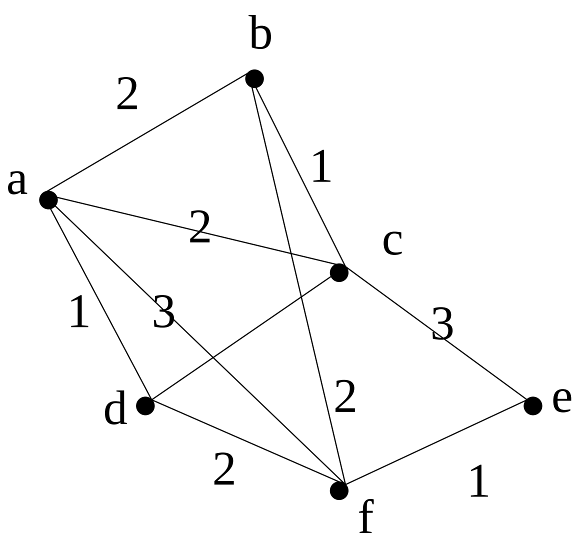
3. Draw a logic network equivalent to the reduced expression in part 2 (3 pts).

Problem 4. Given the adjacency matrix of graph G below:

$$\begin{bmatrix} 0 & 2 & 0 & 1 & 0 & 1 \\ 2 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

1. Draw the graph, using node names a, b, c, d, e, and f. (4 points)
2. Is G simple?
3. Is G connected?
4. Is G complete?
5. Is G planer?
6. Does G have any cycles?
7. What is the degree of each node?

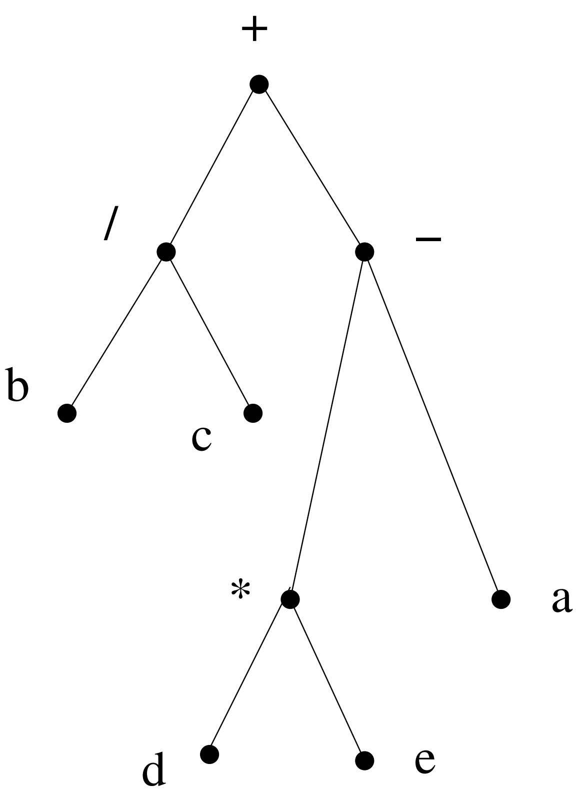
Problem 5. Use Dijkstra's algorithm to find the shortest path between nodes a and e in the following graph. Please show your steps: simply drawing the shortest path will not garner you many points!



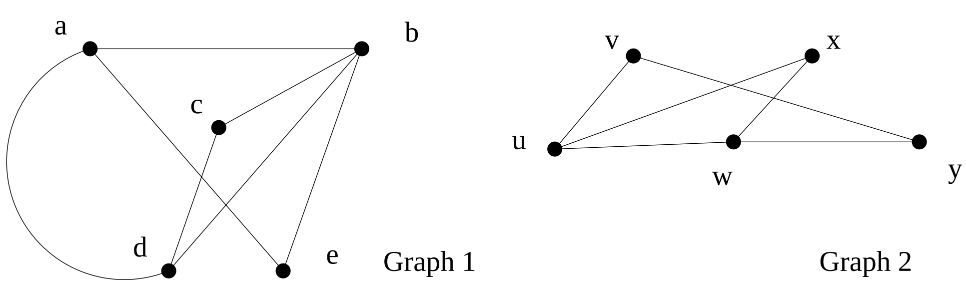
Problem 1. For the tree below, perform the following traversals (you may simply list the results of each traversal next to the corresponding method):

1. inorder
2. preorder

3. postorder
4. general graph depth-first traversal
5. general graph breadth-first traversal

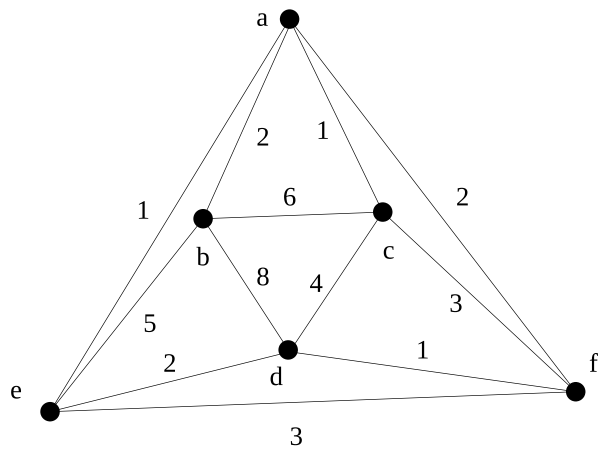


Problem 2. For the following graphs, demonstrate conclusively whether they are, or are not, isomorphic:

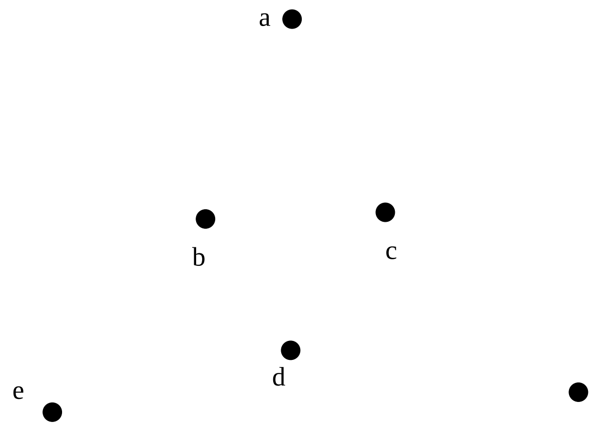


Problem 3. Consider the graph below (called "Original Graph", to the left):

1. Draw a minimal spanning tree for this graph through the vertices next to the graph above. Indicate the total weight of this minimal tree in the space below.



Original Graph



Minimal Spanning Tree

2. Produce an adjacency matrix which represents the graph (with the nodes listed in alphabetical order, to make the grader happy!).

3. Determine whether an Euler path exists. If so, indicate how to trace it, starting from node a.

Problem 4. Using the ten basic properties (1-5a, 1-5b) of a Boolean algebra, and the uniqueness of the identity, prove that

1.

$$(x \cdot y)' = x' + y'$$

2.

$$(x + y)' = x' \cdot y'$$

for any Boolean algebra.

Problem 5. Consider the list (t,h,i,s,w,i,c,k,e,d,t,r,e,e).

1. Create a binary search tree from the list (entered in list order).

2. What is the theoretical worst case number of comparisons for binary tree search using a tree with this many nodes?

3. What is the worst case number of comparisons for binary tree search using this tree?

Problem 6. Consider the set $A = \{m, a, t, h\}$.

1. Is this the same as the set $\{a, t, h, m\}$? Why or why not?

2. Is the set $\{\{a, t\}\}$ a member of the power set of A? Why or why not?

3. Give an example of a binary operation defined on this set.

4. How many distinctly different binary operations can be defined on this set?