

Something to remember :

$a \equiv b \pmod{m}$ and $a \equiv b \pmod{n}$, and
 $\gcd(m, n) = 1$, then

$$a \equiv b \pmod{mn}$$

1. $a \equiv b \pmod{m}$
2. $a \equiv b \pmod{n}$
3. $\gcd(m, n) = 1$

4. $a - b = km$ 1, defn of \equiv
5. $a - b = ln$ 2, defn of \equiv
6. $km = ln$ 4, 5 transitivity of \equiv
7. $n \mid km$ 6, defn of \mid
8. $n \mid k$ 3 + Euclid's lemma
9. $k = nr$ 8, defn of \mid
10. $a - b = rmn$ 4, \equiv
11. $a \equiv b \pmod{mn}$ 10, defn of \equiv