Section 1.1: Statements, Symbolic Representations, and Tautologies

January 9, 2006

Abstract

We encounter the elements of logic: statements, connectives, tautologies, contradictions, etc., and create wffs ("whiffs") from these basic elements. An algorithm for detecting tautologies in the form of implications is described.

- **Statement:** a sentence possessing truth value (T or F).

Exercise #1

a) The moon is made of green cheese. F, a statement

b) He is certainly a tall man.
↑ free variable — Not a statement

c) Two is a prime number T, a statement

d) Will the game be over soon? — not a statement

e) Next year Interest rates will rise. — Statement, but we don't know its truth value.

f)

- **Logical connectives** join statements into **formulas**, or compound statements:
  - conjunction (symbolized by $\wedge$, "and")
  - disjunction (symbolized by $\vee$, "or")
  - implication (symbolized by $\longrightarrow$: *(does its table seem weird to you? It's by convention!)*

    In the implication $A \longrightarrow B$, A is the **antecedent**, and B is the **consequent**. Some English equivalents to implication are given in Table 1.5.

Exercise #4, p 14

4a. Healthy plant growth follows from sufficient water.
sufficient water => healthy plant growth

b. advances => availability

c. errors => modification

d. savings => good insulation or storm windows throughout

and, or, implication

A	B	$A \wedge B$	$A \vee B$	$A \longrightarrow B$	A'	B'	$A \leftrightarrow B$
T	T	T	T	T	F	F	T
T	F	F	T	F	F	T	F
F	T	F	T	T	T	F	F
F	F	F	F	T	T	T	T

Implication plays an especially important role among connectives, so learn it well!

- equivalence (symbolized by $\longleftrightarrow$, "if and only if")
- negation (symbolized by $'$, "not" – *unary*)

Note: These connectives are not independent - some of these may be derived from the others (Exercise #29 shows that conjunction and negation suffice to write the others, for example).

Exercise #6abc

- **Well-formed formula** (wff - "whiff") is a compound statement made up of statements, logical connectives, and other wffs *What makes one well-formed?*
  - **Order of precedence**:
    * parentheses
    * $'$
    * conjunction, disjunction
    * implication
    * equivalence