MAT121 Test 1 (Fall 2007): Functions and Limits

Name:

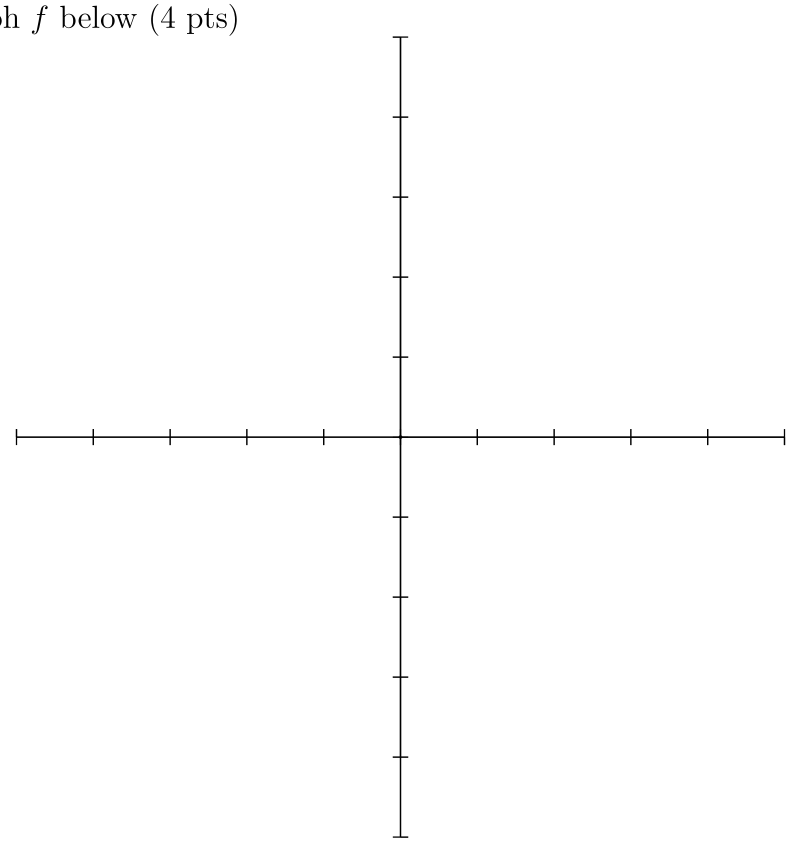
Directions: All problems are equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1. Consider the function f defined by

$$f(x) = \begin{cases} x - b & x < 1 \\ ax + 1 & 1 \leq x < 2 \\ (x-1)^2 + 1 & x \geq 2 \end{cases}$$

a. Choose the parameters a and b so that f becomes a continuous function (6 pts)

b. Carefully graph f below (4 pts)



Problem 2. Consider the functions $f(x) = \sin(x)$ and $g(x) = 1 - x^2$.

a. (2pts) What are the domains and ranges of the two functions?

b. (4pts) Write formulas for $f \circ g$ and $g \circ f$ (make sure to distinguish which is which!).

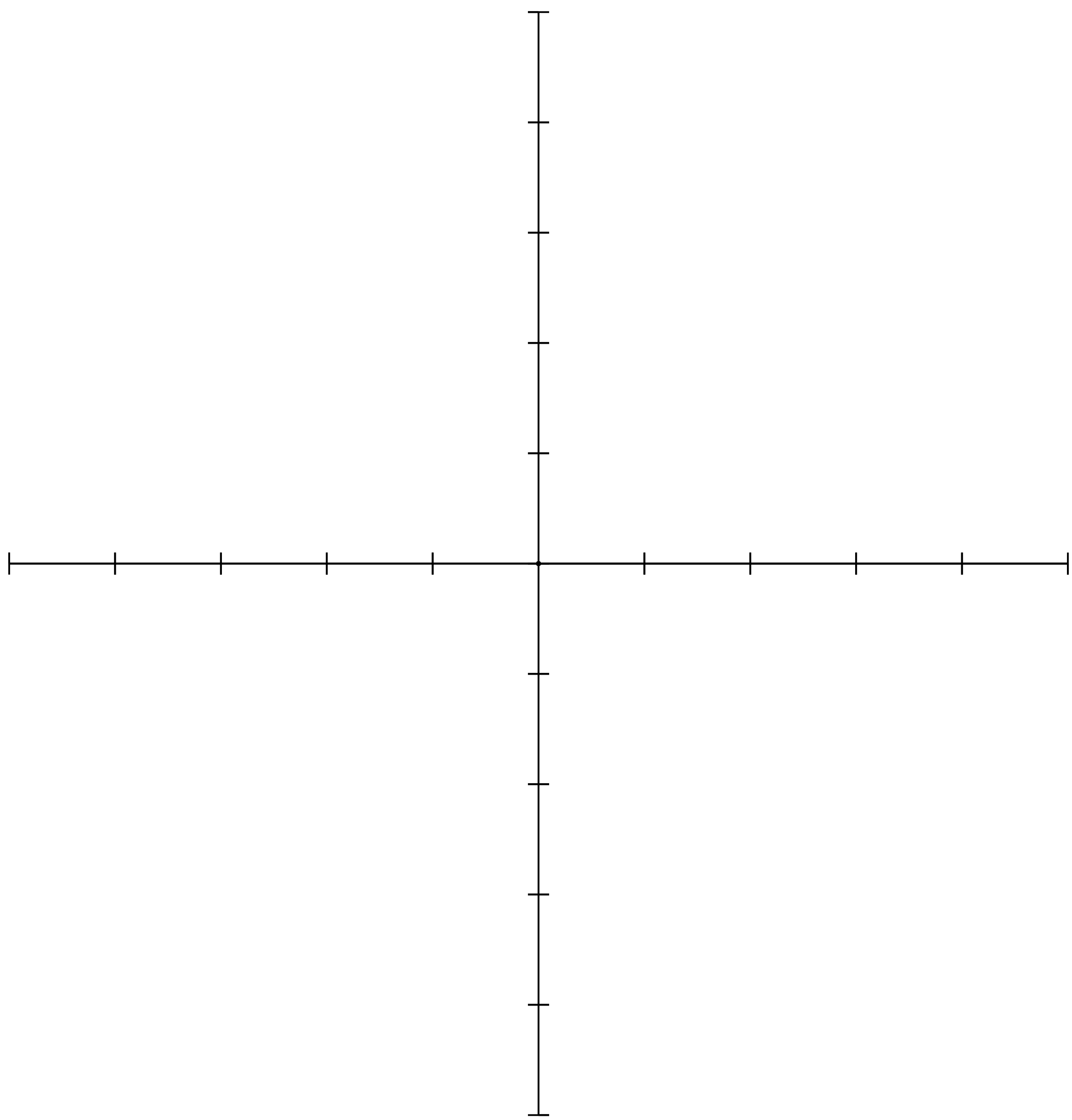
c. (4pts) Find the domains and ranges of $f \circ g$ and $g \circ f$.

d. (2pts) What symmetries do the compositions possess?

Problem 3. Suppose that f is an odd function, and satisfies the following:

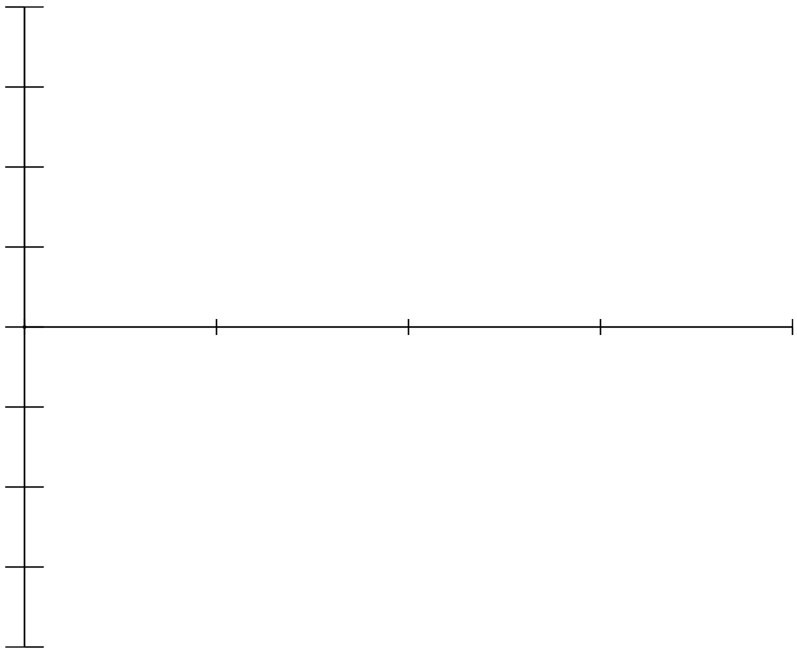
$$\begin{array}{l} f(0) = 0 \text{ and } \lim_{x\to 0^+} f(x) = 1 \\ f(1) = 3 \text{ and } \lim_{x\to 1} f(x) = 2 \\ \lim_{x\to 3^-} f(x) = 1 \text{ and } \lim_{x\to 3^+} f(x) = \infty \\ f(4) = -3 \text{ and } f \text{ is continuous there} \\ f \text{ is continuous elsewhere} \end{array}$$

Carefully draw the graph of a function f consistent with this information on the interval $(-5, 5)$.



Problem 4. Consider the function $f(x) = \cos(x)$. We seek its instantaneous rate of change at $x = \frac{\pi}{4}$.

a. (2pts) Carefully draw f on the interval $[0, \pi]$.



b. (3pts) Using a straight edge, carefully draw three secant lines through the point of interest (the point $(\frac{\pi}{4}, f(\frac{\pi}{4}))$) and through the three points of the graph at

   a. $x = 0$
   b. $x = \frac{\pi}{2}$
   c. $x = \pi$

c. (3pts) Write equations for the three secant lines, as follows:

   a. $x = 0$: in slope-intercept form

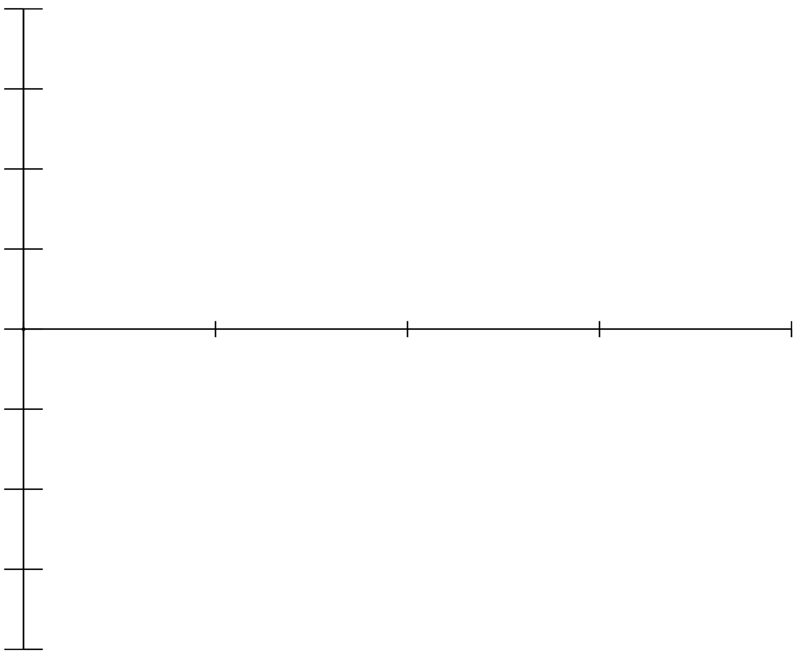
   b. $x = \frac{\pi}{2}$: in point-slope form

   c. $x = \pi$: in point-point form

d. (2pts) Use graphical or numerical means to estimate the slope of the tangent line to the graph at $(\frac{\pi}{4}, f(\frac{\pi}{4}))$: which of the three secant lines gave the best approximation to the instantaneous rate of change?

Problem 5. A math professor leaves his home at 8:00 am ($t = 0$) and drives along a given highway to a distant town. He arrives 10 hours later (at 6:00 pm, $t = 10$). His position was given by the "going" function $g(t) = \sqrt{t/10}$. He stays overnight, then returns home along the same route, leaving at 8 a.m. according to the "coming" function $c(t) = cos(\frac{\pi t}{20})$.

a. (4pts) Depict the situation using g and c in the axes below:

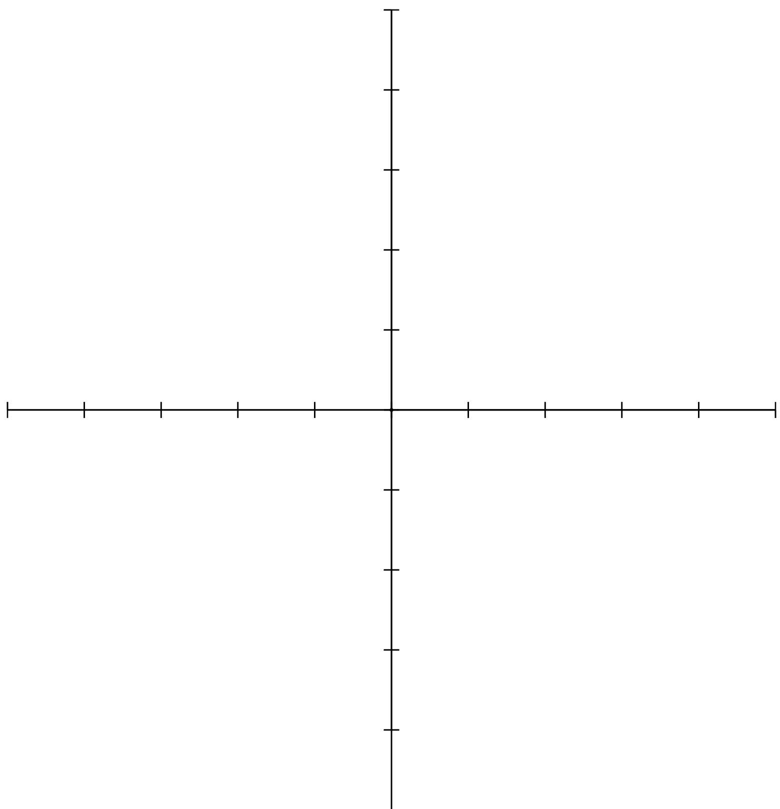


b. (6pts) Demonstrate that there is a point along the two-lane highway at which he was at the same point at the same time of day (in a different lane, however!), just 24 hours apart.

Problem 6. Consider the function

$$f(x) = \frac{x + \sqrt{x+6}}{|x+2|}$$

a. (3pts) Carefully draw the graphs of the functions making up f below: x, $\sqrt{x+6}$, and $|x+2|$.



b. (1pt) What is the domain of f?

c. (6pts) Determine the limit $\lim_{x \to -2^+} f(x)$: you may cite appropriate limit laws, use tricks you've learned, use properties of the classes of functions making up f, etc., but carefully explain how you found the limit, justifying each step. A graphical or numerical solution is better than nothing, but will only get half credit.