MAT122 Final (Fall 2007): Part I: techniques

Name:

Directions: All problems are equally weighted. You **must** skip one: write "SKIP" on it.

Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1. Consider the following definite integral:

$$I = \int_3^5 (x^2 - 2)dx$$

a. (2pts) Use R_4, L_4, M_4, and T_4 (trapezoidal) to approximate I. Use the table below if you like:

Method					Estimate
R_4					
L_4					
T_4					
M_4					

b. (3pts) Express I as the limit of a Riemann sum.

c. (3pts) Evaluate I using the Fundamental Theorem of Calculus.

d. (2pts) Compare the true value with your answers in part (a). Which approximation gave the best result? Do they behave as you expect?

Problem 2. Consider the region bounded by the curves $y = x^2$ and $y = 4x$.

a. (3pts) Carefully sketch this region on the axes provided:

b. (4pts) Find the **exact** area of the region.

c. (3pts) Carefully illustrate **the strategy** behind your calculation of the area either below, or on the graph above. That is, illustrate the various components of the integral.

Problem 3. Consider the function $f(x) = \frac{1}{16}x^2 + 3$ for $x \geq 0$.

a. (2pts) Demonstrate that f is invertible on this interval.

b. (4pts) **Carefully** sketch the graphs of both f and f^{-1} below:

c. (2pts) Find the value of the derivative of the inverse at $x = 4$ (you do not need to actually obtain f^{-1} to do this, if you're clever!).

d. (2pts) Write an integral that expresses the area bounded between the two curves f and f^{-1} without any reference to f^{-1}.

Problem 4. Use L'Hôpital's rule, exponentials, and logs to establish that $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$. [Hint: recall our transformation of the function $x^{1/x}$.]

Problem 5. Consider the function

$$f(x) = \int_1^x \frac{dt}{t}$$

a. (6pts) Use three iterations of Newton's method with initial guess $x_0 = 11$ to find the value of x for which $f(x) = 3$.

b. (4pts) Find the **exact** solution of $f(x) = 3$. Compare to your best approximation using Newton's method.

Problem 6. Perform the following integrations by hand, with all details:

a.

$$A = \int \sin(x) e^{\cos(x)} dx$$

b.

$$B = \int \frac{2x-1}{x^2+1} dx$$

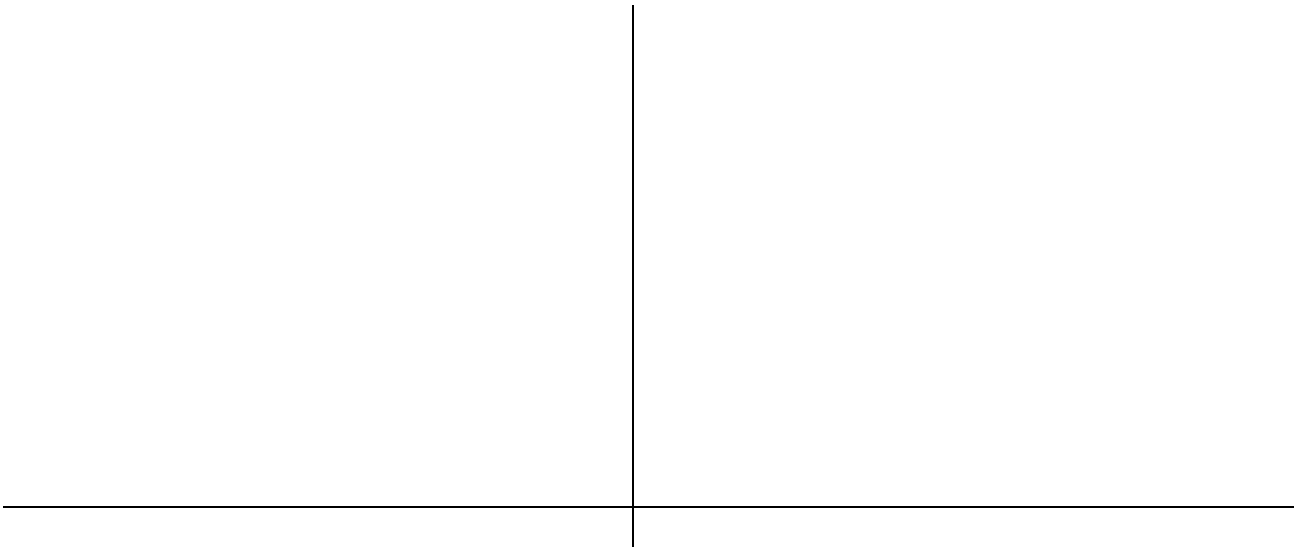
Problem 7. Consider the linearization of $\tan(x)$ at $x = \frac{\pi}{4}$.

a. (8pts) Use this linear approximation of tan to estimate tan(.75) to four decimal places.

b. (2pts) What is the error in the approximation, as a percentage of the true value?

Problem 8. You knew that you'd have to do some sort of volume integration, didn't you? So here it is: consider the function $f(x) = e^x$ on the interval $[0, 1]$, and form a bowl by rotating the area between the graph and the x-axis about the y-axis.

a. (2pts) Graph the function, and a 3-d representation of the bowl.



b. (4pts) Find the volume using Discs/Washers (write (but it's not necessary to evaluate) the integral).

c. (4pts) Find the volume using cylindrical shells (write (but it's not necessary to evaluate) the integral).