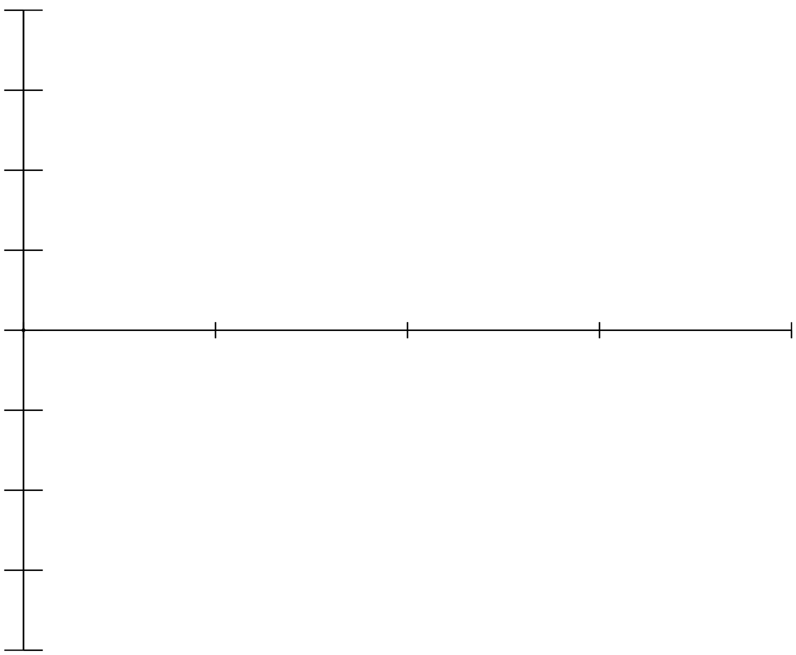
MAT122 Test 1 (Fall 2007): Integration

Name:

Directions: All problems are equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1.

a. Illustrate (pictorially) how to use Newton's method to find a solution of the equation $\sin(x) = \cos(x)$ on the interval $[0, \pi/2]$.



b. Produce the next three iterates starting from an initial guess of $x = 1$ (use all of your calculator's decimals!).

c. How good is your answer by the third iterate?

Problem 2. Use the following rectangle rules to approximate the integral $I = \int_0^4 x^2 dx$:

a. R_4

b. L_4

c. T_4 (trapezoidal)

d. M_4

e. Which gives the best approximation?

Problem 3. Calculate $\int_0^1 x dx$ in two different ways:

a. By sketching the relevant signed area and using geometry.

b. As the limit $\lim_{N\to\infty} R_N$ (remember that $\sum_{j=1}^{N} j = \frac{N(N+1)}{2}$ – brought to you by Gauss!).

Problem 4. Demonstrate how to apply the FTC I (in gory detail) to evaluate the following integral:

$$I = \int_1^9 \frac{t+1}{\sqrt{t}} dt$$

Check your answer with your calculator!

Problem 5.

a. Find an antiderivative of the function $\sin(x^2)$ which is zero at $x = 1$.

b. What is the geometric significance of the function $g(x) = \int_1^x f(t)dt$?

What about $h(x) = \int_1^x |f(t)|dt$?

c. Compute the derivative of

$$g(x) = \int_0^{\sin(x)} \cos(t)dt$$

Problem 6.

a. Replace the following integrals in x with simpler integrals via u-substitutions (do not evaluate the final integrals in u):

i.

$$A = \int x \cos(x^2) dx$$

ii.

$$B = \int x^2 (x^3 + 1)^4 dx$$

iii.

$$C = \int \frac{\cos(\sqrt{x})}{\sqrt{x}} dx$$

b. Evaluate the following by u-substitution:

$$I = \int_0^1 \frac{x}{(3x + 2)^3} dx$$

Problem 7. Compute the area between the two functions $f(x) = x(2-x)$ and $g(x) = 1 - |x-1|$ on the interval $[0, 2]$. Illustrate the area with a sketch.