MAT122 Test 2 (Fall 2007): Applications of integrals; Exponentials

Name:

Directions: All problems are equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

In this exam, we will be using the following function on $[0,20]$ in several problems:

$$v(x) = 40 \left(\frac{2(x-8)}{(x-8)^2 + 16} + 1 \right)$$

Problem 1. Consider the area A bounded by $v(x)$, the x-axis, the y-axis, and the line $x = 20$.

a. (4 pts) On the graphs below, plot views of the volumes that one obtains when one rotates A about the x-axis (V_1, left), and about the line $x = 25$ (V_2, right).
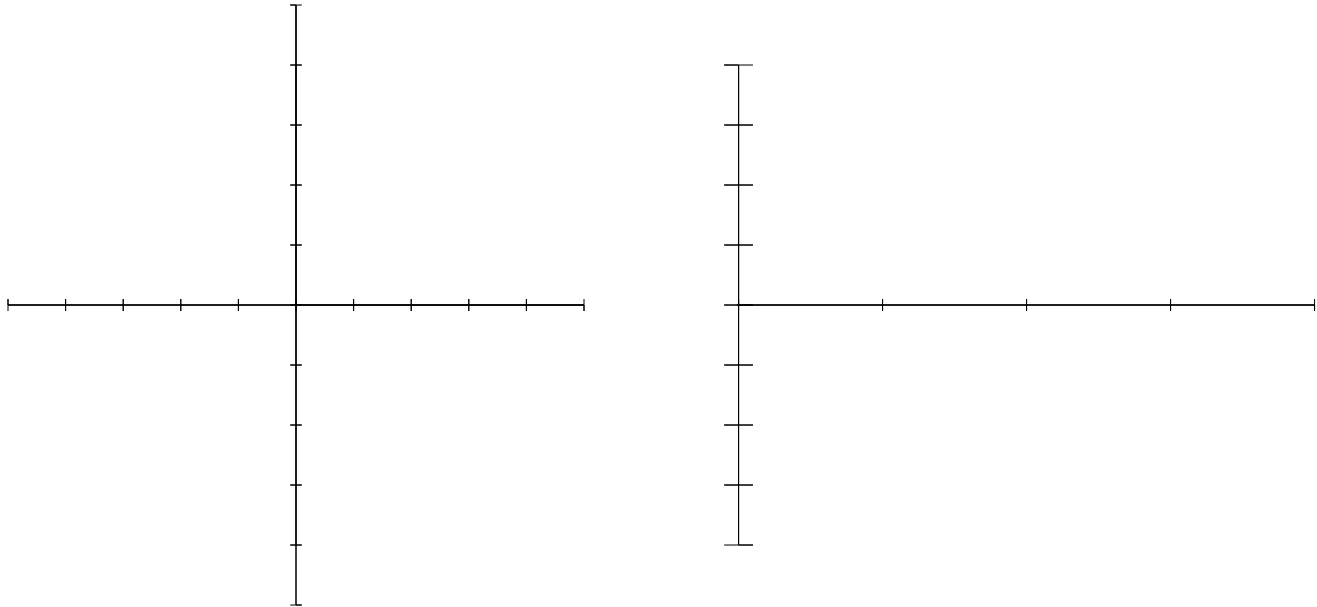
b. (6 pts) Write the volumes V_1 and V_2 as integrals, then evaluate them (use your calculator!).

$V_1 =$

$V_2 =$

Problem 2. According to the National Institute of Standards and Technology

http://ts.nist.gov/WeightsAndMeasures/Metrology/upload/historyof105-8.pdf

the average density of dirt is 120 lbs per cubic foot. [Bet you didn't know that!;)]

A ramp of dirt is constructed in the shape of a wedge on a flat concrete surface. The ramp is 10 feet wide, 100 feet long, and rises 10 feet.

a. (2 pts) Draw a picture that summarizes all this information.

b. (7 pts) How much work is done in lifting dirt from ground level to construct this ramp?

c. (1 pt) What's the volume of the dirt contained in the ramp? [This is easier perhaps than you imagine at first glance!]

Problem 3. An oil spill is radiating out in the ocean in a circular pattern from its source (a sunken ship). The thickness of the oil (in meters) is a function of the distance from the ship (in meters):

$$\rho(r) = \frac{10000}{r^2 + 10000}$$

a. (2 pts) Draw a picture summarizing all details of the situation.

b. (8 pts) Calculate the amount of oil in the area a kilometer in radius around the ship.

Problem 4. Suppose that $v(x)$ represents the average velocity of traffic (in miles per hour) at various points along a road from the center of town (at 0 miles) to a point 20 miles outside the city (refer to your sketch of v from Problem 1).

a. (3 pts) Assume that a car travels at velocity $v(x)$ along this route. **How** could we use this information to **estimate** the time it takes the car to go from mile 8 to mile 9? [Make sure that the units of your answer are in hours.]

b. (3 pts) How much time dt would it take the car to go from mile x to mile $x + dx$?

c. (3 pts) As one drives along from mile 0 to mile 20, signs above the highway report estimated times to mile 20. If you're at mile m, what integral represents the estimated time from mile m to mile 20? [Hint: $T = \int dt$, with appropriate limits.]

d. (1 pts) How much time does it take the car to go from mile 8 to mile 20?

Problem 5. Consider the function $f(x) = e^{2x} - 2x$

   a. (3 pts) Find the critical points of f and determine whether they are local minima, maxima, or neither.

   b. (4 pts) Find the equation of the tangent line to the graph of f at the point where $x = 1$.

      $y =$

   c. (3 pts) Evaluate the indefinite integral $I = \int (e^{2x} - 2x)dx$.

      $I =$