

$$4) a) (x')' = x$$

$$x' + x = x + x' \quad (1a)$$

$$= 1 \quad (5a)$$

and

$$x' \cdot x = x \cdot x' \quad (1b)$$

$$= 0 \quad (5b)$$

Therefore $x = (x')'$ by the theorem on the uniqueness of complements.

$$b) (x+y)' = x' \cdot y', \quad (x \cdot y)' = x' + y'$$

$$(x+y) + (x' \cdot y') = ((x+y) + x') \cdot ((x+y) + y') \quad 3a$$

$$= (x + (y + x')) \cdot (x + (y + y')) \quad 2a$$

$$= (x + (x' + y)) \cdot (x + 1) \quad 1a, 5a$$

$$= ((x + x') + y) \cdot 1 \quad 2a$$

$$= (1 + y) \cdot 1 \quad 5a$$

$$= (y + 1) \cdot 1 \quad 1a$$

$$= 1 \cdot 1 \quad \text{Prac 3a}$$

$$= 1 \quad 4b$$

and

$$(x+y) \cdot (x' \cdot y') = (x' \cdot y') \cdot (x+y) \quad 1b$$

$$= (x' \cdot y') \cdot x + (x' \cdot y') \cdot y \quad 3b$$

$$= x \cdot (x' \cdot y') + (x' \cdot y') \cdot y \quad 1b$$

$$= (x \cdot x') \cdot y' + x' \cdot (y \cdot y) \quad 2b$$

$$= 0 \cdot y' + x' \cdot (y \cdot y') \quad 5b, 1b$$

$$= y' \cdot 0 + x' \cdot 0 \quad 1b, 5b$$

$$= 0 + 0 \quad 3b$$

$$= 0 \quad 4a$$

Now assert the result by the uniqueness of complements.