385 Review

Note Title 12/9/2007

#42, p570

	0	1	Output
0	5	3	1
1	5	2	0
2	1	3	0
3	2	4	1
4	2	0	1
5	1	4	0

0-equivalents

$\{1,2,5\}$, $\{0,3,4\}$

1-equivalents

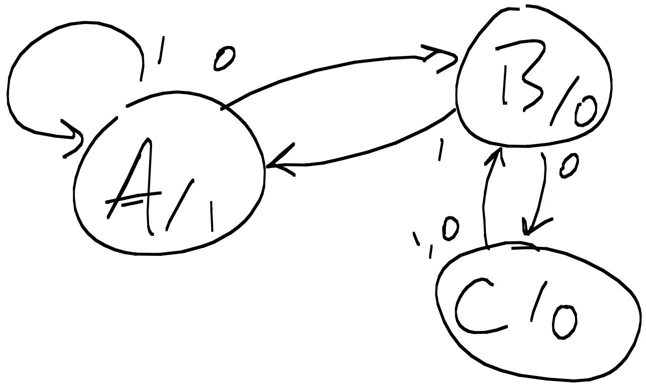
$\{1\}$ $\{0,3,4\}$

$\{2,5\}$

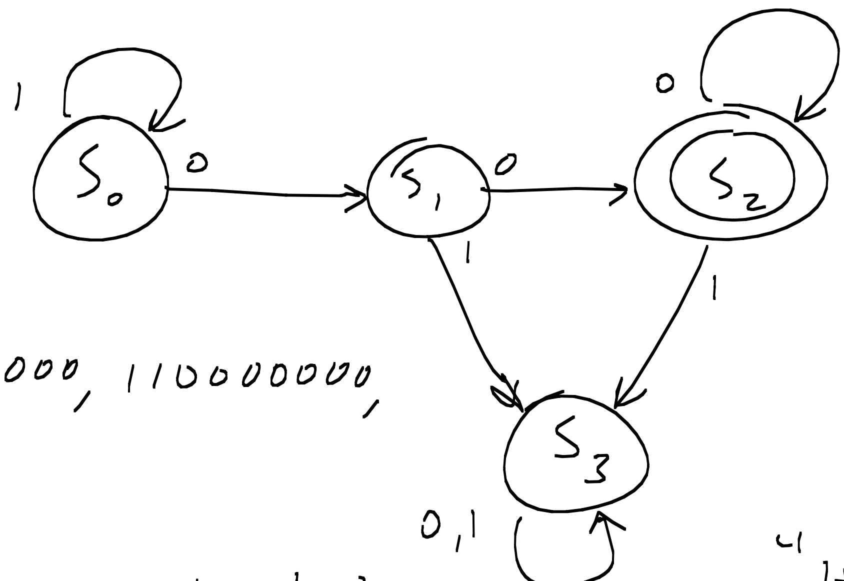
2-equivalents

$\{1\}$, $\{2,5\}$, $\{0,3,4\}$

C, B, A



#26 p 567

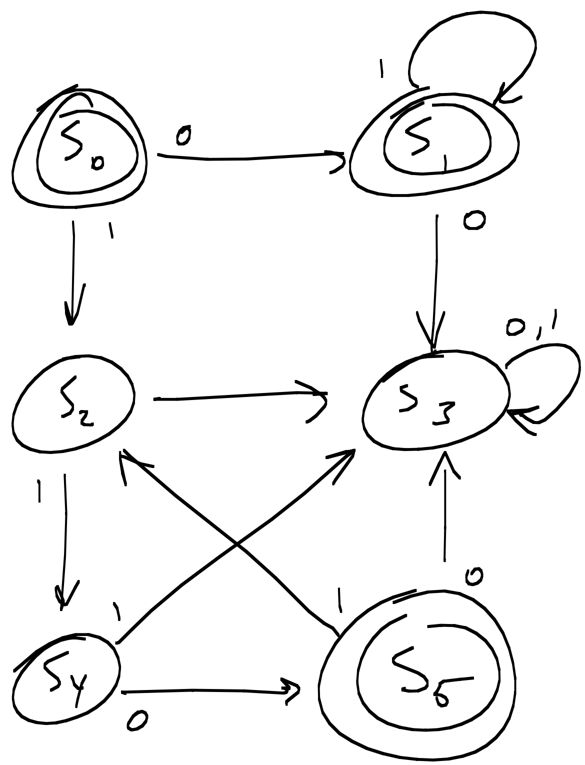


R = {00, 100, 1100, 00000, 110000000, ... }

"1*000*" (equivalent to "1*00*0")

0 is an example of $1^* 0 0^*$, unrecognized.

#27 p 548 (#31, p. 641)



$R = \{ \lambda, 0, 01, 011, 0111, \ldots,$
$110, 110110, 110110110, \ldots \}$

"$(110)^* \vee 01^*$"

#33137 p569

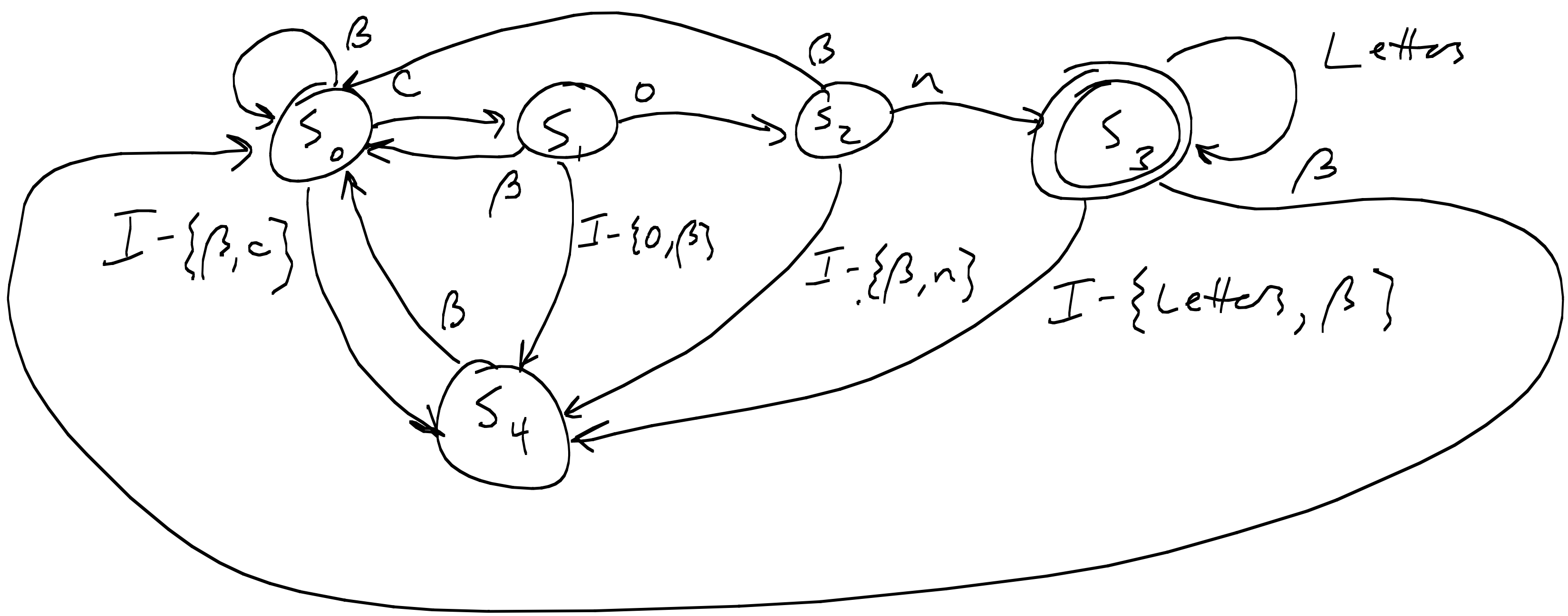
a. 0111 0111 $(1^*01)^*(11 \vee 0^*)$

$01 \in$ "1^*01"

$1101 \in$ "1^*01"

$11 \in$ "$11 \vee 0^*$"

#21/25 p564/640



#26, p 141/142 ^{128}

$$T(0) = 1$$

$$T(1) = 2$$

$$T(n) = 2T(n-1) + T(n-2) \qquad n \geq 2$$

Prove that $T(n) \le (5/2)^n$ for $n \ge 0$

$$T(2) = 2 \cdot T(1) + T(0) = 2 \cdot 2 + 1 = 5$$

$$T(3) = 2 \cdot T(2) + T(1) = 2 \cdot 5 + 2 = 12$$

$$T(4) = 2 \cdot T(3) + T(2) = 2 \cdot 12 + 5 = 29$$

Base case: $T(0) = 1 \le (5/2)^0$ ✓

$T(1) = 2 \le (5/2)^1$ ✓

Assume $P(r)$ $\forall r \in \{0, \dots, k\}$, & show $P(k+1)$.

$P(k)$: $T(k) \le (5/2)^k$

$P(k+1)$: $T(k+1) \le (5/2)^{k+1}$

Consider

$$T(k+1) = 2 \cdot T(k) + T(k-1)$$

$$\leq 2 \cdot \left(\tfrac{5}{2}\right)^{k} + \left(\tfrac{5}{2}\right)^{k-1}$$

$$\leq \underbrace{\left(2 \cdot \tfrac{5}{2} + 1\right)}\left(\tfrac{5}{2}\right)^{k-1}$$

$$\leq \frac{24}{4} \left(\tfrac{5}{2}\right)^{k-1} \leq \frac{25}{4}\left(\tfrac{5}{2}\right)^{k-1} = \left(\tfrac{5}{2}\right)^{k+1}$$

$\therefore P(k+1)$, & the result holds for all n by the second principle of mathematical induction.

#21/23 p 515/590

# of 1s		
four	1 1 1 1	1, 2
three	1 1 1 0	1, 3
	1 0 1 1	2, 4
two	1 0 1 0	3, 4, 5
	0 1 0 1	6
one	1 0 0 0	5, 7
	0 1 0 0	6, 8
zero	0 0 0 0	7, 8

# of 1s		
three	1 1 1 –	1
	1 – 1 1	1
two	1 – 1 0	1
	1 0 1 –	1
one	1 0 – 0	
	0 1 0 –	
zero	– 0 0 0	
	0 – 0 0	

* 1111
* 1110
* 1011
* 1010
* 1000
0000
* 0100
* 0101

# of 1s	
	1 – 1 –

	*	*	*	*	*	*	*	*
	1111	1110	1011	1010	0101	1000	0100	0000
1-1-	✓	✓	✓	✓				
10-0				✓		✓		
010-					✓		✓	
-000						✓		✓
0-00							✓	✓

$$x_1 x_3 + x_1' x_2 x_3' + x_2' x_3' x_4'$$

(same as p506, by Karnaugh)

$\cancel{6}$ 7d   p ~~475~~/548

$$(x+y')\cdot z = [(x'+z')\cdot(y+z')]'$$

$$[(x'+z')\cdot(y+z')]' = (x'+z')' + (y+z')' \quad \text{de Morgan}$$

$$= (x')'\cdot(z')' + y'\cdot(z')' \quad \text{de Morgan}$$

$$= x\cdot z + y'\cdot z \quad \text{double negation}$$

$$= (x+y')\cdot z \quad \text{distributive}$$