Section 7.2: Logic Networks

November 7, 2007

Abstract

We examine the relationship between the abstract structure of a Boolean algebra and the practical problem of creating logic networks for solving problems. There is a fundamental equivalence between Truth Functions, Boolean Expressions, and Logic Networks which allows us to pass from one to the other.

1 An Example Application, and Fundamental Parallels

Example: Two light switches, one light!

The problem is as follows: A light at the bottom of some stairs is controlled by two light switches, one at each end of the stairs. The two switches should be able to control the light independently. How do we wire the light?

- A Truth Function

s_1	s_2	$L(s_1, s_2)$
On	On	On
On	Off	Off
Off	On	Off
Off	Off	On

s_1	s_2	$L(s_1, s_2)$
1	1	1
1	0	0
0	1	0
0	0	1

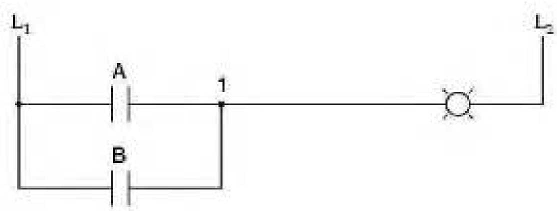
$(s_1 \wedge s_2) \vee (s_1' \wedge s_2')$

$(s_1 \cdot s_2) + (s_1' \cdot s_2')$

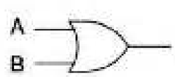
- A Boolean Expression

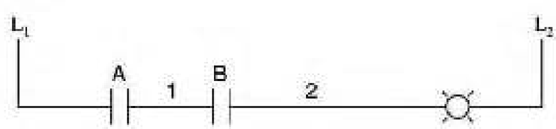
- A Logic Network (Basic Mechanics and Conventions)

  - Input or output lines are not tied together except by passing through gates:
    - OR gate
    - AND gate

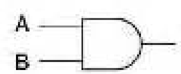


A	B	Output
0	0	0
0	1	1
1	0	1
1	1	1

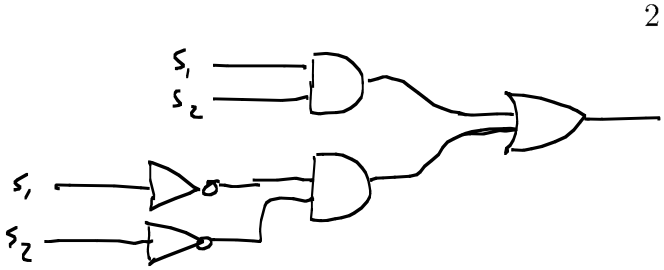


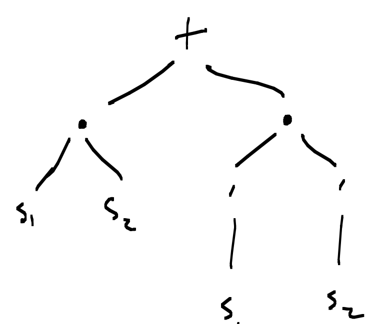


A	B	Output
0	0	0
0	1	0
1	0	0
1	1	1



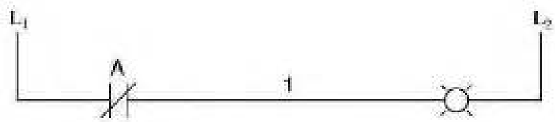
$$(s_1 \cdot s_2) + (s_1' \cdot s_2') = (s_1 \cdot s_2) + (s_1 + s_2)'$$



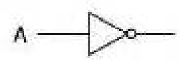


$(+ \cdot s_1\ s_2 \cdot {}'s_1\ {}'s_2)$

* NOT gate



A	Output
0	1
1	0



- Lines can be split to serve as input to more than one device.
- There are not loops with output of a gate serving as input to the same gate (feedback).
- There are no delay elements.

2 Applications

2.1 Converting Truth Tables to Boolean Expressions (Canonical Sum-of-Products Form)

Example: Practice 11, p. 485/558

x_1	x_2	x_3	$f(x_1,x_2,x_3)$
1	1	1	1
1	1	0	0
1	0	1	1
1	0	0	1
0	1	1	0
0	1	0	0
0	0	1	1
0	0	0	1

$f(x_1,x_2,x_3) =$

$x_1 x_2 x_3 +$

$x_1 x_2' x_3 +$

$x_1 x_2' x_3' +$

$x_1' x_2' x_3 +$

$x_1' x_2' x_3'$

$= x_1 x_2 x_3 + x_2'$

$= x_1 x_3 + x_2'$

$x_2' + (x_2')' x_1 x_3$

Example: Exercise 10/11, p. 495/568

x_1	x_2	x_3	$f(x_1, x_2, x_3)$
1	1	1	0
1	1	0	0
1	0	1	1
1	0	0	1
0	1	1	0
0	1	0	1
0	0	1	0
0	0	0	0

$f(x_1, x_2, x_3) =$

$x_1 x_2'(x_3 + x_3') = x_1 x_2'$

$\left\{ \begin{array}{l} x_1 x_2' x_3 + \\ x_1 x_2' x_3' + \end{array} \right.$

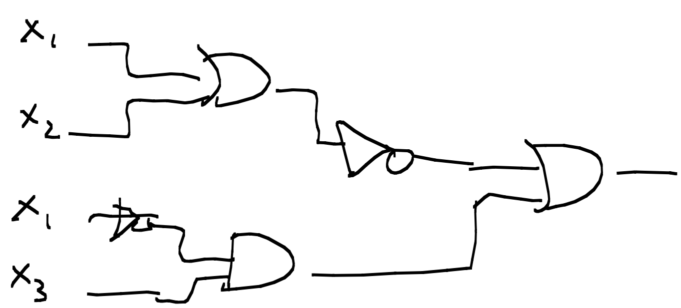
$x_1' x_2 x_3'$

$= x_1 x_2' + x_1' x_2 x_3'$

2.2 Converting Boolean Expressions to Logic Networks

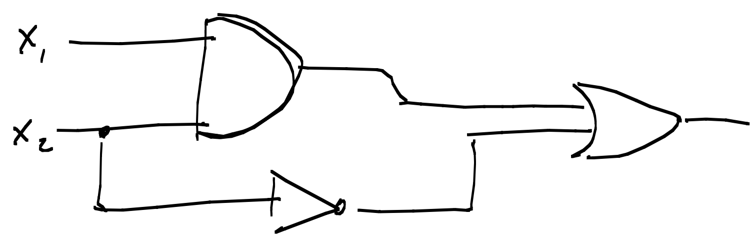
Example: Exercise 1b, p. 493/566

$(x_1 + x_2)' + x_1' x_3$



2.3 Converting Logic Networks to Truth Functions or Boolean Expressions

Example: Exercise 2, p. 493/567



$$x_1 \cdot x_2 + x_2' = f(x_1, x_2)$$

x_1	x_2	$f(x_1, x_2)$
1	1	1
1	0	1
0	1	0
0	0	1

2.4 Simplifying Canonical Form

We can use properties of Boolean algebra to simplify the canonical form, creating a much simpler logic network as a result.

Example: Practice 11, p. 485/558

Full Adder:

c	x_1	x_2	s	c'
1	1	1	1	1
1	1	0	0	1
1	0	1	0	1
1	0	0	1	0
0	1	1	0	1
0	1	0	1	0
0	0	1	1	0
0	0	0	0	0

2.5 Adding Binary numbers

c
x_1
x_2

Half-Adders and Full-Adders

Half-Adder: Adds two binary digits.

x_1	x_2	s	c
1	1	0	1
1	0	1	0
0	1	1	0
0	0	0	0

$$s = x_1'x_2 + x_1x_2'$$
$$c = x_1x_2$$

Note, however, that the half-adder doesn't implement s in this way: instead,

$$s = (x_1 + x_2) \cdot (x_1x_2)'$$

$$= (x_1 + x_2) \cdot (x_1' + x_2')$$
$$= (x_1 + x_2) \cdot x_1' + (x_1 + x_2) \cdot x_2'$$
$$= (x_1 \cdot x_1' + x_2 \cdot x_1') + (x_1 x_2' + x_2 x_2')$$
$$= x_2 x_1' + x_1 x_2'$$

Questions:

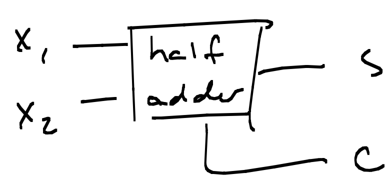
1. How?
2. Why?

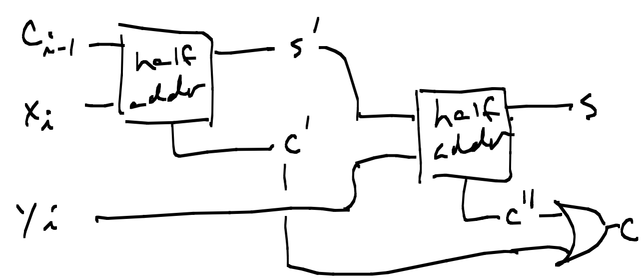
Full-Adder: Adds two digits plus the carry digit (made up of two half-adders, essentially!).

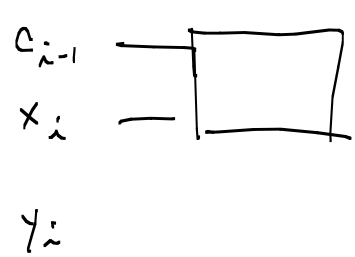
- Give c_{i-1}, x_i, y_i
- Simply use a half-adder to add the carry digit c_{i-1} to the sum digit s of a half-adder of x_i, y_i, to get s_i.
- To get the carry digit c_i, compare carry digits of both half-adders, to see if either gives a 1 (in which case $c_i = 1$).

Example: Practice 12, p. 490/563

Black Box this; we have a new unit called a "half-adder"







$$\begin{array}{r} 11\ \ \\ 101 \\ 111 \\ \hline 1100 \end{array}$$

c	x_1	x_2	s	c
0	1	1	0	1
1	0	1	0	1
1	1	1	1	1