MAT229 Test 1 (Fall 2008): Exponentials, Logs, Inverses, etc.

Name:

Directions: Problems are not equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1. (20 pts) Variety pack – show work!

a. Re-express the function $f(x) = 2^x$ using base e.

b. Solve for x: $\ln(x+1) - \ln(x) = 1$

c. If $\theta = cos^{-1}(x)$, what is $\tan(\theta)$ expressed as a function of x, without trig or arc-trig functions? (Hint: draw a triangle.)

d.

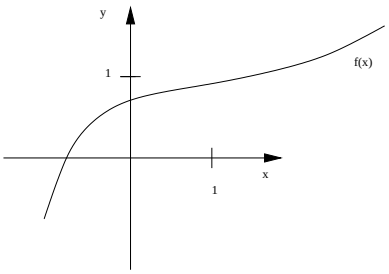


Figure 1: Carefully graph the inverse of the function $f(x)$, whose graph is given here.

Problem 2. (40 pts)

a. (5 pts) Given the function

$$f(x) = x\ln(x) - x$$

(5 pts) Find the extrema of f, and use the second derivative test to decide if the extrema are maxes, mins, or neither.

b. (5 pts) Find the root r of f analytically (show work!).

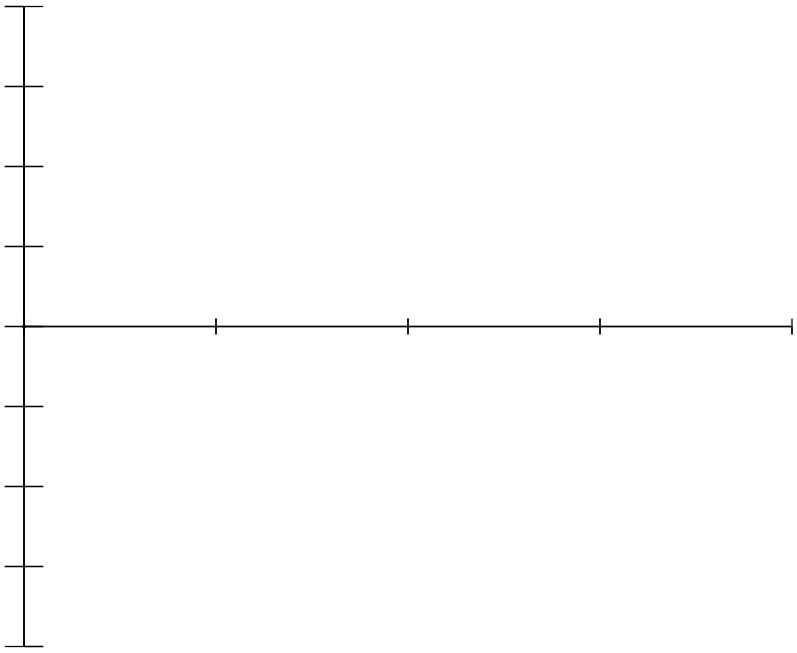
c. (5 pts) Use L'Hôpital's rule to find $\lim_{x\to 0} f(x)$.

d. (5 pts) Find the linearization L of f about the point $x = 2$.

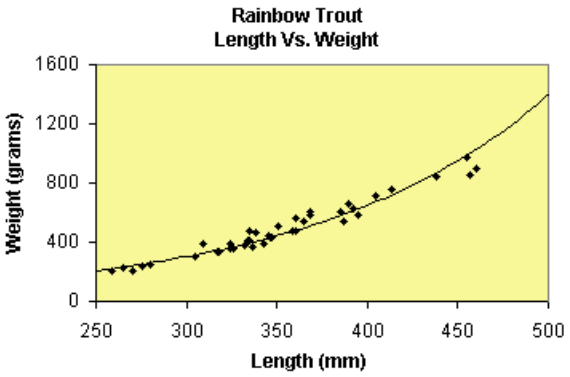
e. (5 pts) Use Newton's method to compute the next two approximations x_1 and x_2 to the root r, starting from an initial guess of $x_0 = 2$. Give exact answers if possible, and approximate answers correct to the thousandth place.

f. (5 pts) How does the value x_1 of Newton's method relate to the linearization of f about the point $x = 2$?

g. (10 pts) Carefully graph f and the linearization L on the interval $(0, 4)$, showing all relevant features from the analysis above.



Problem 3. (20 pts) Rainbow trout taken from 4 different localities along the Spokane River (eastern Washington) during July, August and October of 1999 were analyzed for heavy metals for the Washington State Department of Ecology. As part of this study, the length (in millimeters) and weight (in grams) of each trout were measured: Three data points that appear to lie on the



graph are (247,184), (385,565), and (455, 975).

a. (10 pts) Estimate the exponential rate constant using these data.

b. (5 pts) Do these three points appear to be consistent with an exponential model? Justify.

c. (5 pts) Use these points to estimate an appropriate exponential model $W(l)$.

Problem 4. (20 pts) Suppose that a particle is moving with speed (in miles per second)

$$s(t) = \tan^{-1}(t)$$

starting from $t = 0$.

a. (5 pts) What is the particle's asymptotic speed as $t \to \infty$?

b. (5 pts) At what time will the particle attains 95% of its asymptotic speed? Give your answer exactly and approximately (correct to the thousandth place).

c. (10 pts) How far will the particle have travelled at that point? Give your answer exactly, and approximately (correct to the thousandth place).