MAT229 Test 2 (Fall 2008): Applications of Integrals; sequences; series

Name:

Directions: Problems are not equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1. (15 pts) Consider the integral

$$I = \int_0^1 x^2 dx$$

Given that $\Delta x = 1/4$,

a. fill in the entries in the following table so as to calculate the trapezoidal approximation to the integral I.

Table 1: $T_4 =$

	x_0	x_1	x_2	x_3	x_4
x values					
f values					

b. Same thing, so as to calculate the midpoint approximation to the integral I.

Table 2: $M_4 =$

	x_0	x_1	x_2	x_3
x values				
f values				

c. Show how to compute S_8 from these two estimates. How much error is there in S_8? How do you know?

Problem 2. (15 pts)

a. (10 pts) Use the definition of an improper integral as a limit to calculate the integral

$$I = \int_1^\infty \frac{1}{x^{5/3}} dx$$

Show your work!

b. (5 pts) Use comparison to show that

$$I = \int_1^\infty \frac{1}{x^{5/3} + x} dx$$

converges.

Problem 3. (20 pts) In honor of the Autumn harvest, we construct a cornucopia out of the function

$$f(x) = .5 + \frac{x^3}{1-x}$$

on the interval $[0, .5]$.

a. Draw f on this interval, and write an integral that represents the arc length of this section of the function f; then use your calculator to compute its value.

b. Now write an integral that represents the surface area of rotation of this section of the function f (about the x-axis), and use your calculator to compute its value. Then draw the cornucopia.

Problem 4. (15 pts)
(10 pts) Write the third degree Taylor series polynomial approximation to the function of Problem 3, $f(x) = .5 + \frac{x^3}{1-x}$, about the point $x = .25$. For goodness sake, use your calculator as much as possible!

(5 pts) If the fourth derivative of f is bounded by $0 \leq f^{(4)}(x) \leq 800$ on the interval $[0, .5]$, bound the error we make in using the third degree Taylor polynomial to approximate f on $[0, .5]$.

Problem 5. (10 pts) Evaluate

$$I = \int_1^3 \ln(x) dx$$

analytically (that is, without recourse to your calculator). Show your work!

Problem 6. (15 pts) A kid decides to drink some of his parents' white wine, but doesn't want them to find out. In the beginning (day 0) the bottle contains 10 ounces of wine. The first day (day 1) the kid drinks two ounces, then tops the bottle off with two ounces of water. Clever kid!

The wine is so good that the kid decides to do it again, and again, and again: each day he drinks two ounces of liquid, and replaces that liquid with two ounces of water.

a. What's the concentration of wine (the fraction of wine) at day n, c_n?

b. How much wine will the kid have drunk at day n, w_n? Write a partial sum which represents this quantity, and evaluate it if you can.

c. What's the limit of the concentration of wine in the bottle as $n \to \infty$?

Problem 7. (10 pts) Show that $\sum_{n=1}^{\infty} \sin \frac{1}{n^2}$ is a positive, convergent series. *Hint*: Use the inequality $\sin x \le x$ for $x \ge 0$.

(Extra Credit - 5 pts). Archimedes used a sequence of regular polygons to approximate π (to bound it between two values, and to squeeze it until he knew its value was between 3.1408 and 3.1429):

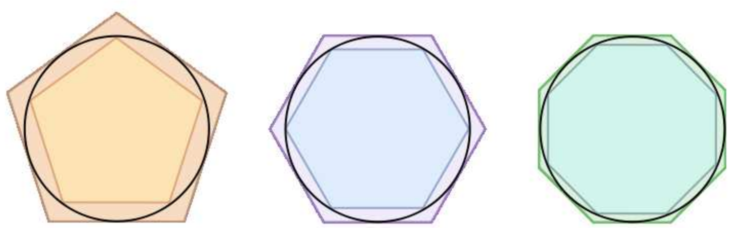

Figure 1: Several successive approximations to π, using regular polygons.

Demonstrate (using this diagram) that

$$\sin(\frac{\pi}{n}) \le \frac{\pi}{n} \le \tan(\frac{\pi}{n})$$

where n is the number of the sides of the polygons used.