MAT229 Test 3 (Fall 2008): Series and Parametric Equations

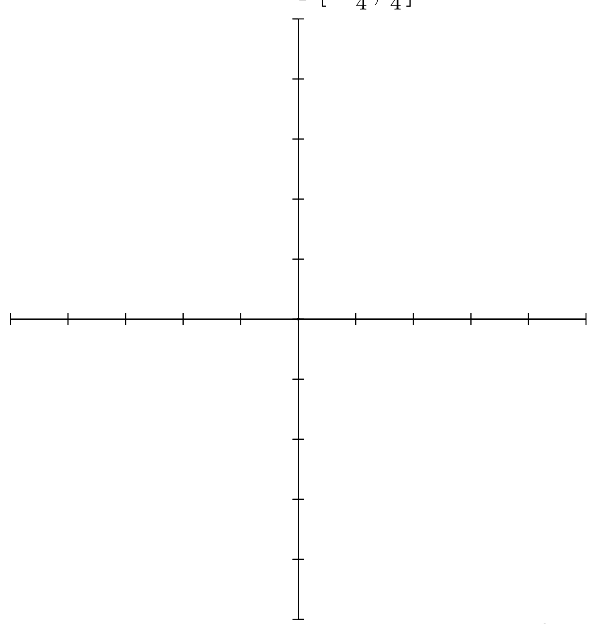
Name:

Directions: Problems are not equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1. (10 pts) The equation of a **lemniscate** is $(x^2 + y^2)^2 = 2a^2(x^2 - y^2)$.

a. (5pts) Find its equation in polar coordinates, using the fact that $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$.

b. (5pts) Plot the lemniscate for $a = 2$ and $\theta \in [-\frac{\pi}{4}, \frac{\pi}{4}]$.



c. What happens if you try to plot the lemniscate from $\theta \in (\frac{\pi}{4}, \frac{3\pi}{4})$?

Problem 2. (15 pts) A particle follows motion described by the curve $c(t)$, in time $t \in \left(0, \frac{\pi}{2}\right)$ (in seconds; lengths are in meters):

$$c(t) = (2\cos(t) - \cos(2t), 2\sin(t) - \sin(2t))$$

a. (5pts) Find the speed of the particle at time $t = \frac{\pi}{4}$.

b. (5pts) Find the time(s) at which the particle's path is vertical in the $xy-$plane.

c. (5pts) Find the length of the particle's path over the interval $t \in \left(0, \frac{\pi}{2}\right)$. Use some preliminary simplification, but use your calculator when/if you need to.

Problem 3. (10 pts) For the particle whose trajectory is given by

$$c(t) = (t^2 - 9, t^2 - 8t)$$

a. (5pts) Find the equation of the tangent line at $t = 2$.

b. (5pts) Find the time(s) where the tangent line has slope 0.

Problem 4. (20 pts)

a. (5pts) Give an example of a series which is conditionally convergent but not absolutely convergent.

b. (7pts) Use the root test to show that the series

$$\sum_{n=0}^{\infty} 3^{-n}$$

is absolutely convergent. Then compute the value of the series (show work – don't just give a numerical answer).

c. (8pts) Determine convergence or divergence of the following series using any method:

$$S = \sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln(n))^3}$$

Problem 5. (20 pts) The MacLaurin series for arctan is given by

$$\tan^{-1}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$$

a. (5pts) Find the radius of convergence of this series, and demonstrate in particular that this series is valid when $x = 1$.

b. (8pts) Differentiate this series term-by-term; by inspection of the resulting series, deduce a closed-form expression for the derivative of arctan.

(Problem 5, cont.)

c. (7pts) We can use this series to estimate the value of π. $\tan^{-1}(1) = \frac{\pi}{4}$. How many terms must you take to assure that the approximation you get by truncating the Taylor series is within .0001 of the correct value of π? (**2pts extra credit: give the approximate value of π using this number of terms.)**

Problem 6. (10 pts) Find the Taylor series expansion of the function $f(x) = x^4 + 3x^3 - 2x^2 + x - 2$ about $c = 2$.

Problem 7. (15 pts) Use power series to find the solution of the differential equation $f'(x) = 2f(x)$, where $f(0) = e$. That is, assume a solution of the form

$$P(x) = \sum_{n=0}^{\infty} a_n x^n$$

and determine the a_n.