# MAT225 Test 1 (Spring 2008): Chapter 1

**Name:**

**Directions**: Each problem is worth 10 points. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

## Problem 1

a. (6 pts) Find the solution set of the following system: $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{bmatrix} 3 & 1 & 1 \\ 2 & 2 & 1 \\ 1 & 3 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

b. (4 pts) Give a complete geometric description of the set.

## Problem 2

a. (8 pts) Determine all values of $h$ such that the following three vectors are linearly dependent:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 4 \\ -7 \end{bmatrix}, \qquad \mathbf{v}_2 = \begin{bmatrix} -2 \\ 5 \\ 3 \end{bmatrix}, \qquad \mathbf{v}_3 = \begin{bmatrix} 8 \\ -7 \\ h \end{bmatrix}$$

b. (2 pts) Describe the geometry in the case of dependence: how does it compare to the geometry in the case of a choice of $h$ leading to independence?

**Problem 3** Illustrate each of the following terms or concepts with a well-chosen example:

a. Homogeneous linear system

b. span of a set of vectors

c. linear dependence of a set of vectors

d. basic and free variables

e. inconsistent linear system

**Problem 4** If oxygen and hydrogen are combined, water is produced (and an explosion occurs – but that's another story). The chemical equation looks something like this:

$$H_2 + O_2 \longrightarrow H_2O$$

But there is a problem: there are two oxygen atoms on the left, and only one on the right. We need to balance this equation. Consider each compound as a vector of hydrogen atoms and oxygen atoms, and then

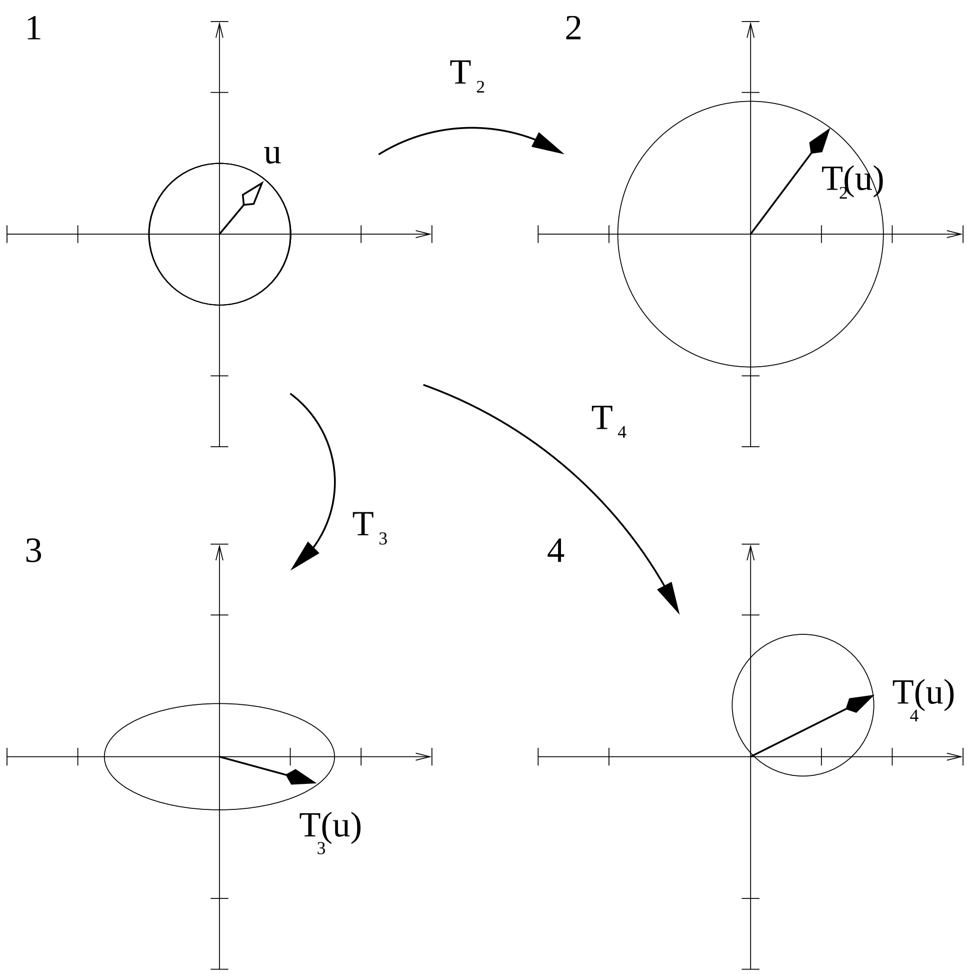
a. Write a linear system that accounts for the conservation of hydrogen and oxygen atoms, making sure that there are the same number on each side of the chemical equation.

b. Then solve the system to balance the equation (check that your solution works!). How many solutions are possible? What kinds of solutions make sense?

**Problem 5** The following figure purports to show three different linear transformations of the space $\Re^2$. In the upper left corner is the unit circle, and the vector $\mathbf{u}$ that rests on one point of the unit circle. The other three panels show purported linear transformations, and the image of the vector $\mathbf{u}$ under each transformation. Your tasks:

a. (7 pts) Decide whether any of the three panels 2-4 could conceivably represent a linear transformation of the plane in panel 1. Could panel 1 represent the result of a linear transformation on itself?

b. (3 pts) Draw the image of $-\mathbf{u}$ under those transformations which are conceivably linear.



**Problem 6** Consider the following problem, the likes of which you might have encountered in high school:

$$\begin{array}{rcrcrcr} & & 2x_2 & + & 2x_3 & = & 4 \\ 3x_1 & + & 2x_2 & + & 2x_3 & = & 7 \\ x_1 & - & 4x_2 & + & 2x_3 & = & -1 \end{array}$$

**Without solving the system**, describe at least two distinctly different perspectives that linear algebra provides on the solution of this problem, using the notation and vocabulary of chapter 1 in your descriptions.

a. Perspective 1:

b. Perspective 2:

c. Perspective 3:

## Problem 7

a. (8 pts) Now solve the system of Problem 6 using the reduced row echelon form, with partial pivoting, and showing each step. You may scale the rows to get a leading 1 **before** proceeding from row echelon to reduced row echelon form.

b. (2 pts) Why is it unnecessary for you to do partial pivoting, if using infinite precision arithmetic (that is, keeping all decimals of every number in the process)? Why should a computer program to solve general linear systems do partial pivoting?