MAT225 Test 2 (Spring 2008): Chapter 2 and Section 4.1

Name:

Directions: Problems are equally weighted (10 points). Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1

a. (5 pts) Let $A = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -4 \\ -6 & k \end{bmatrix}$. For what values of k (if any) will $AB = BA$?

b. (5 pts) Suppose that

$S : \mathbb{R}^3 \to \mathbb{R}^2$ has matrix representation $A = \begin{bmatrix} 0 & -1 & 2 \\ 4 & 11 & 2 \end{bmatrix}$

and

$T : \mathbb{R}^2 \to \mathbb{R}^3$ has matrix representation $B = \begin{bmatrix} 3 & -1 \\ 1 & 2 \\ 6 & 1 \end{bmatrix}$.

Find a matrix representation for the composition $(S \circ T) : \mathbb{R}^2 \to \mathbb{R}^2$.

Problem 2

a. (3 pts) Explain the process by which the inverse of an invertible $n \times n$ matrix is calculated.

b. (6 pts) Use this process to compute the inverse of $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & 1 \end{bmatrix}$

c. (1 pts) Check your answer.

Problem 3 Let $A_{n\times n}$ be invertible.

a. (3 pts) What can we say about the solution set of $A\mathbf{x} = \mathbf{b}$?

b. (3 pts) What can you can say with certainty about the image of the n-dimensional ball centered at the origin of $\mathbb{R}^n$ under the linear transformation $T : \mathbb{R}^n \to \mathbb{R}^n$ given by $T(\mathbf{x}) = A\mathbf{x}$?

c. (4 pts) Suppose that AB is invertible. Demonstrate that B is invertible.

Problem 4

a. (4 pts) Compute the LU decomposition of the matrix $A = \begin{bmatrix} 1 & -2 & 5 \\ 2 & -3 & 6 \\ 3 & -2 & 0 \end{bmatrix}$ (ignore pivoting).

b. (4 pts) Show how to use it to compute the solution of $A\mathbf{x} = \mathbf{b}$, where $\mathbf{b} = \begin{bmatrix} -2 \\ -3 \\ -2 \end{bmatrix}$.

c. (2 pts) Show how to find the solution of $A\mathbf{x} = \mathbf{b}$ ($\mathbf{b}$ as above) a lot faster!

Problem 5 For A and B as in Problem 1.b.,

$$A = \begin{bmatrix} 0 & -1 & 2 \\ 4 & 11 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 & -1 \\ 1 & 2 \\ 6 & 1 \end{bmatrix}$$

a. (2 pts) Compute BA.

b. (4 pts) Represent BA as the sum of a pair of outer products of vectors.

c. (4 pts) BA is the matrix of a linear transformation T. Tell me all that you can about T.

Problem 6 Let $M_{2\times 2}$ be the vector space of all 2×2 matrices.

a. Let H_1 be the collection of 2×2 matrices of the form

$$\left\{ \begin{bmatrix} x & (x+1) \\ 0 & 0 \end{bmatrix} : x \in \mathbb{R} \right\}$$

Show that H_1 <u>is not</u> a subspace of $M_{2\times 2}$.

b. Let H_2 be the collection of 2×2 matrices of the form

$$\left\{ \begin{bmatrix} x & (x+y) \\ 0 & 0 \end{bmatrix} : x \in \mathbb{R} \right\}$$

Show that H_2 <u>is</u> a subspace of $M_{2\times 2}$.