MAT225 Section Summary:
1.2: row reduction and echelon forms

1. **Definitions**

   - **echelon matrix**
     a matrix in echelon form
   - **pivot position**
     a component of matrix A corresponding to a leading 1 in the reduced echelon form of A. A **pivot** is the value in the pivot position.
   - **pivot column**
     a column of A that contains a pivot position
   - **basic variables**
     variables corresponding to pivot positions
   - **free variables**
     variables other than basic variables

2. **Theorems/Formulas**

   Each matrix is row equivalent to one and only one reduced echelon matrix.

   A linear system is consistent if and only if the rightmost column of the augmented matrix is not a pivot position.

3. Properties/Tricks/Hints/Etc.

   Gaussian elimination (using partial pivoting) for Row reduction:

   (a) Begin with the leftmost column. This is a pivot column, with pivot position at the top. Select largest entry (in terms of magnitude) in the pivot column, and interchange rows to move this entry into the pivot position.

(b) Use row replacement to create zeros in all positions below the pivot.

(c) Iterate, using the submatrix below the pivot row and to the right of the pivot column.

(d) Beginning with the rightmost pivot and working left, create zeros above each pivot.

(e) Scale all pivot rows by the pivot value, so that pivots are 1.

Using row reduction to solve a linear system:

(a) Write the augmented matrix of the system

(b) Use the row reduction algorithm to obtain an equivalent augmented matrix in echelon form. Decide whether the system is consistent. If so, continue; else stop.

(c) Continue row reduction to obtain the reduced echelon form.

(d) Write the system of equations corresponding to the matrix in reduced echelon form.

(e) Rewrite each nonzero equation from step 4 so that its one basic variable is expressed in terms of any free variables appearing in the equation.

4. Summary

In this section we consider an algorithm (Gaussian elimination with partial pivoting) for reducing a system of equations to reduced echelon form, which is unique and allows us to write the solution set of a linear system. The solution may contain free parameters, meaning that there are infinitely many solutions.