MAT225 Section Summary: 4.4

Coordinate Systems

Summary

A basis gives us a way of writing each vector $\mathbf{v}$ in a vector space in a unique way, as a linear combination of the basis vectors. The coefficients of the basis vectors can be considered the coordinates of $\mathbf{v}$ in a coordinate system determined by the basis vectors.

Theorem 7: the Unique Representation Theorem

Let $B = \{\mathbf{b}_1, \ldots, \mathbf{b}_n\}$ be a basis for a vector space V. Then for each $\mathbf{x}$ in V, there exists a unique set of scalars $c_1, \ldots, c_n$ such that

$$\mathbf{x} = c_1\mathbf{b}_1 + \ldots + c_n\mathbf{b}_n$$

Coordinates: Suppose $B = \{\mathbf{b}_1, \ldots, \mathbf{b}_n\}$ is a basis for V, and $\mathbf{x}$ is in V. The **coordinates of x relative to the basis** B are the weights $c_1, \ldots, c_n$ such that $\mathbf{x} = c_1\mathbf{b}_1 + \ldots + c_n\mathbf{b}_n$.

$$[\mathbf{x}]_B = \begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix}$$

is the **coordinate vector of** x **(relative to** B), or the
B**-coordinate vector of** x. The mapping

$$\mathbf{x} \mapsto [\mathbf{x}]_B$$

is the **coordinate mapping (determined by** B).

Example: #1, p. 253

Let

$$P_B = [\mathbf{b}_1\ \mathbf{b}_2\ \cdots\ \mathbf{b}_n]$$

Then

$$\mathbf{x} = P_B[\mathbf{x}]_B$$

is the link between the standard basis representation of $\mathbf{x}$ (on the left) and the representation of $\mathbf{x}$ in the basis B.

Example: #5, p. 254

Example: #14, p. 254

Theorem 8: Let $B = \{\mathbf{b}_1, \ldots, \mathbf{b}_n\}$ be a basis for a vector space V. Then the coordinate mapping $\mathbf{x} \mapsto [\mathbf{x}]_B$ is a one-to-one linear transformation from V onto $\mathbb{R}^n$.

This is an example of an *isomorphism* ("same form") from V onto W. These spaces are essentially indistinguishable.

Example: #23, p. 254

Example: #24, p. 254