MAT225 Section Summary: 4.5
The Dimension of a Vector Space
Summary

Theorem 9: If a vector space V has a basis $B = \{\mathbf{b}_1, \ldots, \mathbf{b}_n\}$, then any set in V containing more than n vectors must be linearly dependent.

Theorem 10: If a vector space V has a basis of n vectors, then every basis of V must consist of exactly n vectors.

dimension of a vector space: If V is spanned by a finite set, then V is **finite-dimensional**, and the dimension V ($\dim V$) is the number of vectors in a basis for V. The dimension of the zero vector space $\{\mathbf{0}\}$ is defined to be zero. If V is not spanned by a finite set, then V is said to be **infinite-dimensional**.

Example: #2, p. 260

Theorem 11 Let H be a subspace of a finite-dimensional vector space V. Any linearly independent set in H can be expanded, if necessary, to a basis for H. Also, H is finite-dimensional and

$$\dim H \leq \dim V$$

Example: #11, p. 261

Theorem 12 (the Basis Theorem): Let V be a p-dimensional vector space, $p \leq 1$. Any linearly independent set of exactly p elements in V is automatically a basis for V. Any set of exactly p elements that spans V is automatically a basis for V.

Example: #22, p. 261

Let A be an m x n matrix. Then the dimension of Nul A is the number of free variables in the equation $A\mathbf{x} = \mathbf{0}$, and the dimension of Col A is the number of pivot columns in A.

Example: #14, p. 261

Example: #27, 28, p. 262