MAT225 Section Summary: 4.6
Rank
Summary

Rank: The rank of a matrix is the dimension of the column space of A. That is, it is equal to the number of independent vectors among the columns of the matrix.

row space: the row space of a matrix A is the span of the rows of A.

Theorem 13: If two matrices A and B are row equivalent, then their row spaces are the same. If B is in echelon form, the non-zero rows of B form a basis for the row spaces of A and B.

Theorem 14 (The Rank Theorem): The dimensions of the column space and the row space of an m x n matrix A are equal (the rank of A). The rank satisfies the relation

$$\text{rank } A + \dim \text{ Nul } A = n$$

You may be wondering why the Nul space popped up here: the point is that all these spaces are fundamentally connected.

Example: #2, p. 269

The Invertible Matrix Theorem (continued): Let A be an n x n matrix. Then the following statements are each equivalent to the statement that A is an invertible matrix:

- The columns of A form a basis of $\mathbb{R}^n$.
- Col $A = \mathbb{R}^n$
- dim Col A = n
- rank A = n
- Nul $A = \{\mathbf{0}\}$
- dim Nul $A = 0$

Examples: #5,8-11, p. 269

Example: #16, p. 269

Example: #18, p. 270

Example: #24, p. 270