$P_1 \wedge P_2 \to Q \qquad (P_1 \wedge P_2 \wedge Q' \to 0) \to Q$

1. $n \in \mathbb{N}$	hyp
2. $n > 2$	hyp
3. $n^2 - 1$ prime	in pursuit of contradiction
4. $n^2 - 1 = (n+1)(n-1)$	3, factoring
5. $(n+1=1) \vee (n-1=1)$	3, defn. of prime
6. $(n=0) \vee (n=2)$	5, number facts.
7. $(n=0)' \to (n=2)$	6, imp
8. $(n=0)'$	1.
9. $n=2$	7, 8, mp
10. $n \neq 2$	2
11. 0	9, 10, contradiction

$$\left[P_1 \wedge P_2 \wedge Q' \to 0\right] \leftrightarrow \left[P_1 \wedge P_2 \to (Q' \to 0)\right]$$

$$\leftrightarrow \left[P_1 \wedge P_2 \to (Q \vee 0)\right]$$

$$\leftrightarrow \left[P_1 \wedge P_2 \to Q\right]$$

Even so, here's the soln. manual:

57. Proof: $n^2 - 1 = (n+1)(n-1)$ where $n - 1 > 1$, which is a non-trivial factorization, so the number is not prime.

(57) For n a positive integer, $n>2$, n^2-1 is not prime.

Proof: If n is even, $n=2q$ for $q \in \mathbb{Z}$ and $q>1$ since $n>2$. Then $n^2-1=(2q)^2-1 = 4q^2-1=(2q-1)(2q+1)$. Since $2q>2$, $2q-1 \geq 2$ and $2q+1>3$. Thus neither $2q-1$ or $2q+1$ equals 1 so n^2-1 is not prime when n is even. If n is odd, $n=2r+1$ for $q \in \mathbb{Z}$ and $r \geq 1$ since $n \in \mathbb{Z}$ and $n>2$. Then $n^2-1=(2r+1)^2-1=4r^2+4r+1-1=4r^2+4r=4(r^2+r)$. Since $r \geq 1$ $r^2+r \geq 2$ so n^2-1 is not prime. Therefore for $n \in \mathbb{Z}$ and $n>2$, n^2-1 is not prime.

Interesting!

(64) The product of two irrational numbers is irrational.

Counterexample: $\sqrt{2}$ is irrational. The $\sqrt{2} \cdot \sqrt{2} = 2$, which is rational.

(65) The sum of a rational number and an irrational number is irrational.

Proof by contradiction: Assume the sum of a rational number and an irrational number is rational. Let $\frac{p}{q}$ and $\frac{r}{s}$ be rational numbers w/ $p,q,r,s \in \mathbb{Z}$ and $qs \neq 0$. Let x be an irrational number. Then $\frac{p}{q} + x = \frac{r}{s} \Rightarrow x = \frac{r}{s} - \frac{p}{q} = \frac{rq-sp}{sq}$.

This is a contradiction since $rq-sp$ and sq are integers and $sq \neq 0$. Thus the sum of a rational number + an irrational number is irrational