MAT228 Test 1 (Fall 2009):

Name:

Directions: Problems are **not** equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1. (21 pts) Consider the following sequences and their limits. Determine whether the sequences converge or diverge. If a sequence converges then find its limit, with justification.

a. $$a_n = \frac{n}{\sqrt{n^2+1}}$$

b. $$b_n = \frac{3+5^n}{2+4^n}$$

c. $$c_n = \sqrt{n+1} - \sqrt{n-1}$$

Problem 2. (20 pts) We have the following data on the percentage f of citizens unemployed in a certain country over a four-year period:

	t_0	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8
Year	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4
	$f(t_0)$	$f(t_1)$	$f(t_2)$	$f(t_3)$	$f(t_4)$	$f(t_5)$	$f(t_6)$	$f(t_7)$	$f(t_8)$
Percentage	16	11	12	9	12	11	20	19	18

We want to estimate $I = \int_0^4 f(x)dx$ using a numerical integration rule.

(10 pts) For which of the following values of N can you actually calculate LRR_N, RRR_N, T_N, and M_N, given only the data in the table above? Calculate the permissible values, so as to complete the following table (insofar as possible):

N	h	LRR_N	RRR_N	T_N	M_N
1					
2					
3					
4					
5					
6					
7					
8					

(5 pts) What do you notice about the estimates as N varies?

(5 pts) Compute the Simpson's approximations below for admissible values of T_N and M_N (show your work!):

N	Simpson's S_N

Problem 3. (21 pts) Consider the following improper integrals:

a. **Demonstrate** that this integral converges using limits (and integration by parts), and give its value:

$$I = \int_0^\infty xe^{-x}dx$$

b. **Demonstrate** that this integral converges using limits, and give its value:

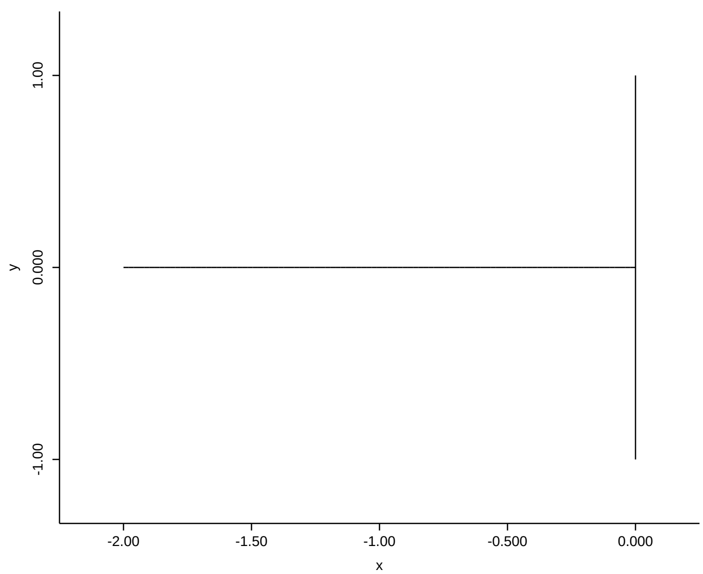
$$I = \int_1^3 \frac{x^2}{\sqrt{x^3-1}}dx$$

c. Decide whether this integral converges by comparison:

$$I = \int_1^\infty \frac{x}{1+x^3}dx$$

Problem 4. (20 pts total) Show how to get the quadratic Taylor polynomial approximation to the function of $f(x) = xe^x$ about $x = -1$. (It's not enough to simply write it down, although that's better than nothing....)

(6 pts) **Carefully** plot f and the Taylor polynomial T_2 on the axes below, over the interval $[-2, 0]$.



(6 pts) The third derivative of f satisfies $.2 \le f^{(3)}(x) \le 1.6$ on the interval $[-1.5, -.5]$.

a. Bound the error we make in using T_2 to approximate f on this interval, and

b. Bound the error that we make at the particular point $x = -1.2$.

Problem 5. (18 pts) Consider the integral

$$I = \int_0^{\pi} x \sin(x) dx$$

(5 pts) Compute the following approximations to fill the following table (you may use your calculator):

Method	estimate
LRR_{20}	
RRR_{20}	
T_{20}	
M_{20}	
S_{20}	

(3 pts) Which method does the best job? Do the results seem reasonable?

For each of the following rules, find the value of N so that the numerical approximation scheme approximates it with an error of at most 10^{-4}, then compute the approximation using your value of N:

a. (5 pts) Trapezoidal rule

b. (5 pts) Simpson's rule

Formulas:

a. $$|T_n(x) - f(x)| \leq K \frac{|x-a|^{n+1}}{(n+1)!}$$

b. $$Error(S_N) \leq \frac{K_4(b-a)^5}{180N^4}$$

c. $$Error(T_N) \leq \frac{K_2(b-a)^3}{12N^2}$$

d. $$Error(M_N) \leq \frac{K_2(b-a)^3}{24N^2}$$