MAT228 Test 2 (Fall 2009): Series

Name:

Directions: Problems are **not** equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1. (20 pts) Determine whether the following series converge absolutely, converge conditionally or diverge:

a. $\sum_{k=1}^{\infty} \frac{\cos(\pi k)}{\sqrt{k}}$

b. $\sum_{n=1}^{\infty} \left(\frac{-n}{2n+3}\right)^n$

Problem 2. (30 pts) Determine whether the following series converge or diverge. Be sure to state the test you use and the appropriate conclusion.

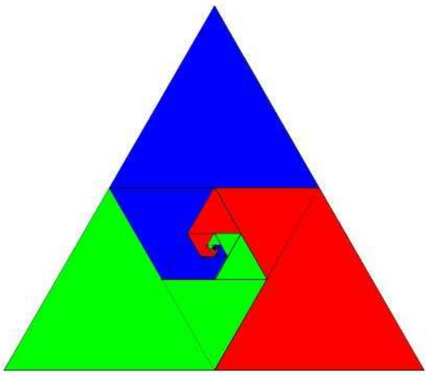
a. $\displaystyle\sum_{k=0}^{\infty} \frac{\sqrt{6k}}{k^{\frac{3}{2}}+5}$

b. $\displaystyle\sum_{k=1}^{\infty} ke^{-k^2}$

c. $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n}{3n+1}\right)$

Problem 3. (10 pts) Do just **one** of the following two problems (write “skip” on other):

a. The following geometric puzzle, which we discussed in class, illustrates the sum of a series, formed using a succession of equilateral triangles. The figure is created by coloring each



“corner equilateral triangle” a different color, and then repeating the process on the central equilateral triangle. This is an example of a recursive process, which will go on forever. However, the sum of the the areas lightest color’s equilaterial triangles is obviously a finite number, representing the total light area in the figure.

Construct an appropriate series equal to the light area, and demonstrate that its sum is 1/3 (of the total area), as we can see from the figure.

b. A ball is dropped from a height of 2 meters, and it bounces until coming to rest. If the properties of the ball are such that the ball returns to a height equal to **half** of its dropped height on each bounce, how far will the ball have travelled before coming to rest?

Problem 4. (10 pts) Find the Taylor series for $f(x) = \ln(x)$, expanded about $c = 1$. The calculator must not be your (only) method – show work. Guess its radius of convergence (with reason!), or else calculate it.

Problem 5. (10 pts) Choose **one** of the following (write "skip" on other):

a. Use the geometric series to find the Taylor series for

$$f(x) = \frac{-2x}{3 - x^2}$$

expanded about $c = 0$. For what values of x is the series valid?

b. Find the values of x for which the following power series converges: $\displaystyle\sum_{n=2}^{\infty} \frac{x^n}{\ln(n)}$