MAT360 Section Summary: 1.1 Review of Calculus

1. Definitions

There are several definitions from calculus presented, including the $\epsilon-\delta$ definition of a limit, definition of continuous function (at a point and on an interval), limit of a sequence, differentiable function (at a point and on an interval), and the Riemann integral.

In conjunction with the Taylor series, the following are also defined:

- n^{th} Taylor polynomial $P_n(x)$ and its associated remainder term $R_n(x)$ (or truncation error);
- Maclaurin series

2. Theorems/Formulas

Theorems exposed are

- the equivalence of continuity at x_0 and limits of convergent sequences at x_0;
- differentiability implies continuity;
- Rolle's Theorem;
- Mean Value Theorem (a generalization of Rolle's);
- Generalized Rolle's Theorem (which just goes to show that there's more than one way to generalize a result!);
- Extreme Value Theorem;
- Weighted Mean Value Theorem for integrals (a generalization of the Mean Value Theorem for integrals);
- Intermediate Value Theorem (note: in their statement, they implicitly specify their understanding of the word "between": they mean "strictly between", otherwise the theorem fails for $f(x) = x^2$ when $a = -1$ and $b = 1$); and
- Taylor's Theorem.

3. Properties/Tricks/Hints/Etc.

Errata: having implied that when they say "between" they mean "strictly between", they operate in #26 as though "between" means "or equal to".

4. Summary

A lot of our time in this course will be spent with our old friend calculus, as we deal with issues that we handled in other ways back in the day.... Now we will ask questions such as how to integrate functions which are not integrable using elementary functions; how to talk about derivatives of functions defined only by data points; how to find roots of functions iteratively; etc.

As examples of intractable functions that we glossed over in calculus class, consider the following problems:

- Compute
$$\int_a^b e^{-t^2} dt$$
- Find the roots of an arbitrary quintic polynomial.

Both of these problems are intractable using methods from your elementary calculus; however, we can use the calculus to get an answer to either **to whatever precision is required** – that's the job of numerical analysis!