MAT360 Exam 1 (Fall 2009)

Name:

Directions: Problems are not equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1. (10 pts) Use three digit rounding and three digit chopping to evaluate

$$\frac{\frac{13}{14} - \frac{6}{7}}{2e - 5.4}$$

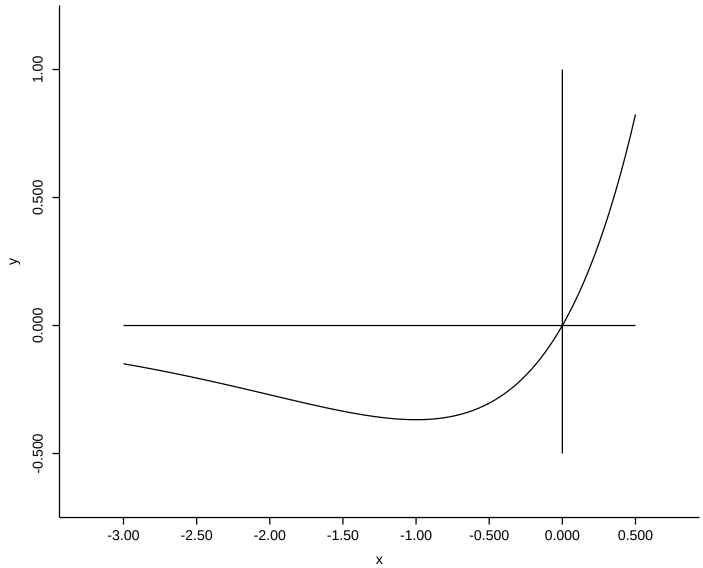
Compute absolute and relative errors and give your final answer in the following table (and show your work below the table):

method	Approximation	Absolute error	Relative Error
3-digit chopping			
3-digit rounding			

- chopping:

- rounding:

Problem 2. (20 pts) Consider the following graph of the function $f(x) = xe^x$:



Find four different starting points for Newton's method with qualitatively different behavior for the iteration (labelling each on the graph, and illustrating using the graph or the space below as you wish):

a. A point where Newton's method converges monotonically to the root;

b. A point where Newton's method will converge non-monotonically;

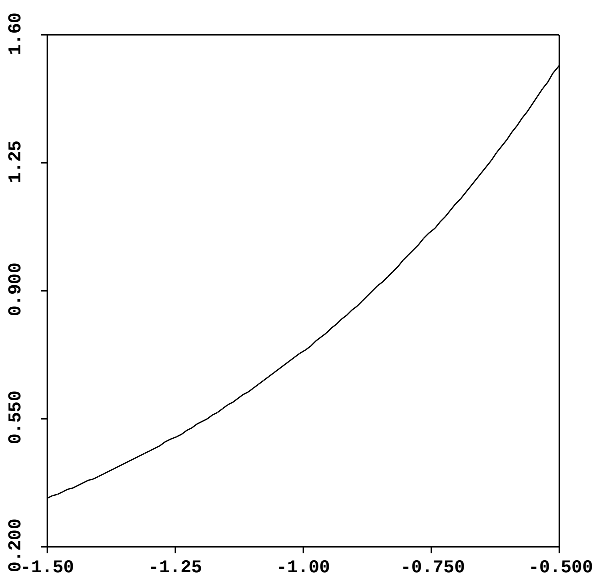
c. A point where Newton's method will "blow up"; and

d. A point where Newton's method will monotonically flee the root.

Problem 3. (10 pts) To which root of $f(x) = (x+2)(x+1)x(x-1)(x-2)$ will bisection converge, if we start with the interval $[-\sqrt{2}, \pi/2]$? How many bisections must you see before you know that? (Justify your answer!)

Problem 4. (10 pts) Demonstrate how to construct the quadratic Taylor polynomial approximation to the function of $f(x) = xe^x$ about $a = -1$.

The third derivative of f is plotted below on the interval $[-1.5, -.5]$ (above left). Bound the error we make in using the quadratic Taylor polynomial to approximate f on this interval.



Recall:

$$|T_n(x) - f(x)| \leq K\frac{|x-a|^{n+1}}{(n+1)!}$$

Problem 5. (20 pts) Consider the following pair of potential fixed point functions $g(x)$ to solve for the root of $f(x) = e^x - \frac{1}{x^2}$:

$$g_1(x) = -2\ln(|x|)$$

$$g_2(x) = e^{\frac{-x}{2}}$$

a. Verify that they are, indeed, fixed point functions for this problem.

b. Compare them, with analysis rather than numerical calculations, and rank them by their convergence if we begin in the neighborhood of the root (around $r = 0.703467422$).

Problem 6. (10 pts) Do **one** of the following two problems:

a. Determine the rate of convergence of $f(h) = \sin(h^2)/h$ to its limit as $h \to 0$. The more justification you can provide, the better!

b. Suppose p is a zero of multiplicity m of f, where $f^{(m)}$ is continuous on an open interval containing p. Show that the following fixed point method has $g'(p) = 0$:

$$g(x) = x - \frac{mf(x)}{f'(x)}$$