MAT360 Exam 2 (Fall 2009): 2.6, 3.1-3.4, 4.1

Name:

Directions: Problems are not equally weighted. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1. (10 pts) Use Taylor series expansions (with error term) to derive the 3-point forward difference formula (using points x_0, $x_0 + h$, and $x_0 + 2h$) for the derivative value $f'(x_0)$. Make sure to justify your consolidation of the error terms.

Problem 2. (30 pts) The **rest of the exam** concerns the following data, generated by the function $f(x) = -x\cos(\pi x)$:

x	f(x)
$x_0 = 0$	0
$x_1 = 1$	1
$x_2 = 2$	-2
$x_3 = 3$	3
$x_4 = 4$	-4

a. (5 pts) Represent the quartic interpolator of the data, using the Lagrange polynomial form. Do not simplify!

b. (10 pts) Use a finite difference table to write the polynomial $p_4(x)$ in Newton's form. Verify that your polynomial interpolates the data.

c. (5 pts) Bound the error you would expect at $x = \frac{3}{2}$ from the interpolating quartic $p_4(x)$, assuming that $f(x)$ generated the data.

d. (10 pts) Compute the portion of a Hermite cubic spline which uses the knots at $x = x_1$ and $x = x_2$, and compute the actual error at $x = \frac{3}{2}$.

Problem 3. (10 pts) Suppose that you want to use a natural (free) cubic spline to fit the data. Write the system of equations that you would solve (in the form $Ax = b$) if each of your splines were written in the form

$$S_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3$$

with $i = 0, 1, 2, 3$. Do not attempt to solve the system! [Hint: don't forget that $h = x_{i+1} - x_i = 1$, which makes life a little easier.]

Problem 4. (10 pts) Suppose that you want to use Muller's method to find a root of the function $f(x) = -x\cos(\pi x)$, starting from the estimates at x_1, x_2, and x_3 (from the table).

Write the interpolating quadratic, and use it to give the next iterate for the true root. Which potential next iterate would you choose, and why?