MAT360 Exam 2 (Fall 2009): Part II, 2.6, 3.1-3.4, 4.1

Name:

Directions: This is the numerical complement to your in-class exam. You are to work alone. For the most part, you will be verifying your answers from the in-class exam in this second part, which will be worth 1/3 of your grade for this exam. You may do Problem 1 on this page. As for the others, you should provide a neat printout of the responses to each, each on separate pages. I would like these returned by Friday's class, 11/20.

Problem 1. (10 pts) The **rest of the exam** concerns the following data, generated by the function $f(x) = -x\cos(\pi x)$: Use the best schemes available in Section 4.1 to estimate the derivative values

x	$f(x)$	$f'(x)$
$x_0 = 0$	0	
$x_1 = 1$	1	
$x_2 = 2$	-2	
$x_3 = 3$	3	
$x_4 = 4$	-4	

at the knots in the table. Show your work below, and put the value (rounded to the fourth decimal place) in the table above.

Problem 2. (20 pts)

a. Find the interpolating quartic $p_4(x)$ to the data in the table. Bound the error you would expect at an arbitrary $x \in (x0, x4)$, assuming that $f(x)$ generated the data. Plot the actual error against the bound.

b. Find the Hermite cubic spline for this data. Bound the error at an arbitrary $x \in (x0, x4)$ and plot the actual error against the bound.

Problem 3. (10 pts) **Use linear algebra** to find the **clamped** cubic spline to fit the data. Write and solve the system of equations given that each of your splines is written in the form

$$S_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3$$

with $i = 0, 1, 2, 3$. Plot the spline and f over the interval $[x0, x4]$.

Problem 4. (10 pts) Use Muller's method to find a root of the function $f(x) = -x\cos(\pi x)$, starting from the estimates at x_1, x_2, and x_3 (from the table).

In choosing between the two roots of the quadratic, take the root closest to the current best estimate. As a stopping criterion take both $|r_n - r| < 10^{-6}$ and $f(r_n) < 10^{-6}$.