

# MAT115 Final Exam (Fall 2005)

Name:

**Directions:** Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it).

Work exactly 10 of the 12 problems. Write “skip” on the other two. You must skip two! Don't do them all and suppose that I'll drop the two lowest....

**Good luck!**

## Problem 1.

1. What is unusual about the Möbius band?
2. What happens when you cut it down the middle?
3. What is the result of cutting it along an edge, starting  $1/3$  of the way in from that edge?
4. Can you think of a useful application one can make of a Möbius band?



**Problem 3.** The authors claim that we've studied "some of the greatest ideas of humankind": pick one, and develop a description, emphasizing the mathematics. [You should be able to fill this entire page, without excessive use of adjectives and/or "large font".]

**Problem 4.** A standard deck of 52 cards is comprised of four suits of 13 cards each (spades, hearts, diamonds, and clubs). What is the smallest number of cards one must draw in order to guarantee that the hand contains

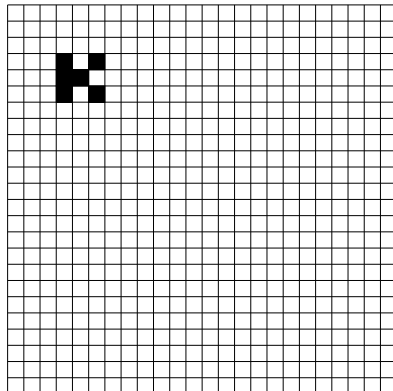
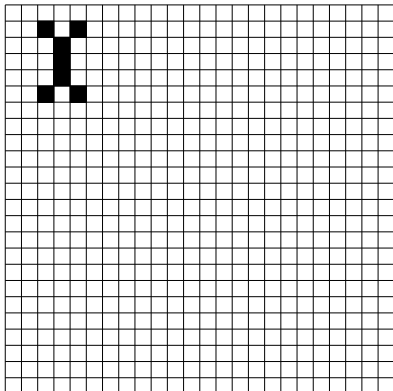
1. three hearts?

2. One card of each suit?

**Problem 5.** In as many ways as possible, describe an octahedron: what is it, what makes it special, what does it look like, etc. Definitely draw it!

**Problem 6.**

1. Given the following initial populations in the game of life, draw the next three generations in the space adjacent to each (rule summary: exactly three living cells adjacent, a dead cell comes alive; below two, or more than three, living neighbors, a living cell dies):



2. Your solutions should exhibit symmetry: explain the type of symmetry you would expect from each.

**Problem 7.** If you've ever listened to the show "Car Talk" on NPR, then you might recognise this as this week's "Car Talk Puzzler":

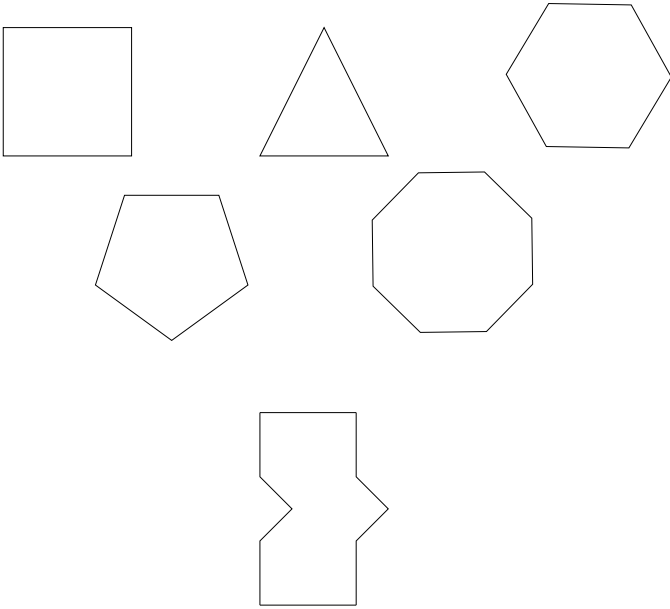
Last year, Tommy got a raise of 10 percent. Ray got a pay cut of 10 percent. This year, Ray got a 10 percent raise, and Tommy got a 10 percent cut. Who's making more money now, and why?

**Problem 8.** Suppose that Fibonacci Nim is to be played starting with 20 sticks, by two wily characters, Sue and Bob, each of whom knows the winning strategy.

1. If Sue goes first, and Bob goes second, who is guaranteed a win if each plays well? (Explain!)

2. Write a plausible sequence of plays, assuming that each player is trying to win by whatever means possible. That is, each player should be making smart moves!

**Problem 9.** Which of the following shapes can be used to tile the entire plane exclusively using that tile? [For those which cannot be used, put an “x” in the middle, and explain why not.]



**Problem 10.** True/false

1.  ( ) Simple interest is better than compound interest in the long term.
2.  ( ) The even integers are more numerous than the odds, because 0 is also even.
3.  ( ) In “Let’s Make a Deal”, you should definitely switch.
4.  ( ) Irrational numbers are those formed with radicals, such as  $\sqrt{81}$ .
5.  ( ) If a boy hates you infinity times, and you hate him infinity+1 times, you hate him more.



**Problem 11.**

1. A coin is flipped 17 times, and comes up heads every time: what is the chance of getting heads on the next toss? (You must explain your answer, and argue against any alternative!).
2. You're playing Monopoly against an unsavory character: three times in a row they roll "snake eyes" with their two dice (that is, each die comes up 1). Are you suspicious? What is the probability of such a strange occurrence? In order to determine it, figure out the following:
  - How many different ways can one roll two ordinary, six-sided dice?
  - How many different ways can one roll doubles?
  - How many different ways to throw two dice three times in succession?
  - How many different ways can one throw the same doubles three times in succession?

**Problem 12.** Suppose that it is currently 3:00p.m. What time will it be

1. in 17 hours?

2. in 177 hours?

3. in 1777 hours?

4. in 17777 hours?

5. in 177777 hours?

If it is Monday today, what day will it be in 49,723 days (and will you care?)?