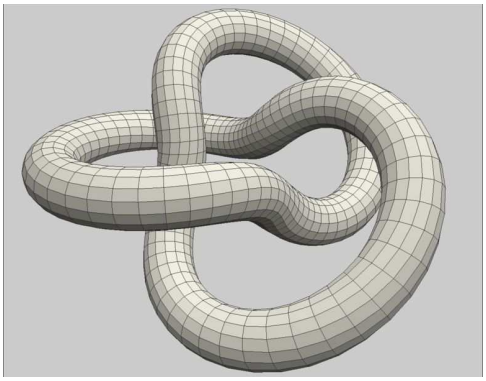
MAT115 Final Exam (Spring 2008)

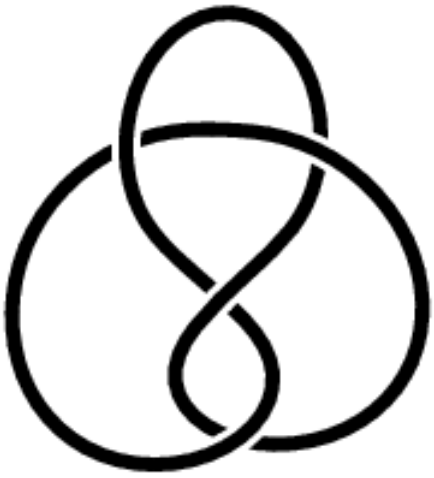
Name:

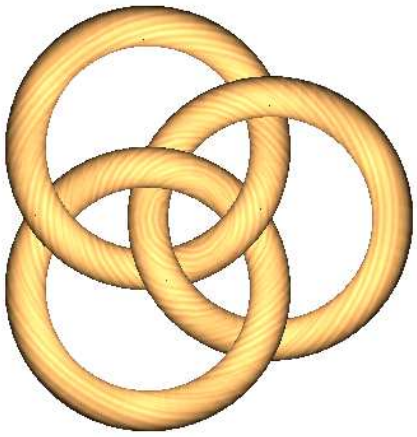
Directions: Show your work: answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Put a box around your final answer to each problem.

Work exactly 10 of the 11 problems. Write "skip" on the other problem. You must skip one! Don't do them all and suppose that I'll drop the lowest.... **Good luck!**

Problem 1. Examine the three figures below, and describe the significance of each one:

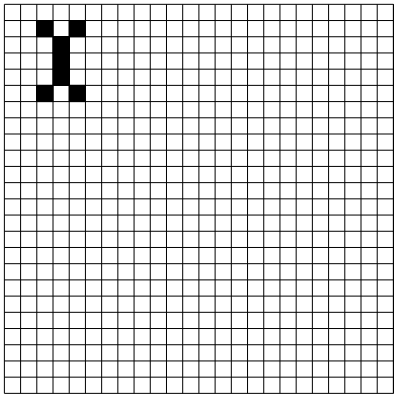


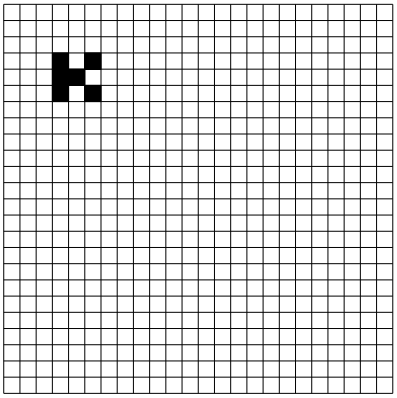




Problem 2.

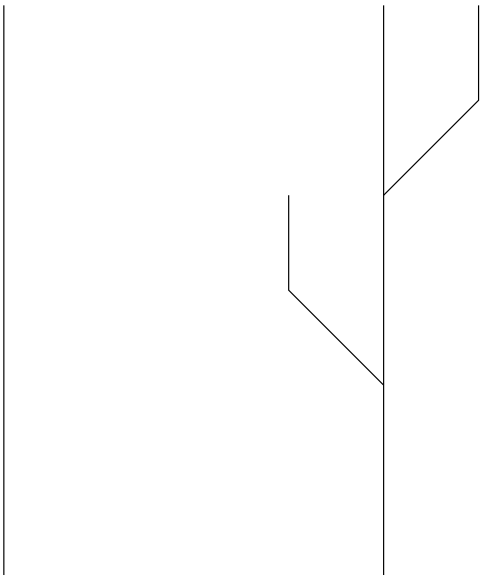
1. Given the following initial populations in the game of life, draw the next three generations in the space adjacent to each:





2. Your solutions should exhibit symmetry: explain the type(s) of symmetry you would expect from each pattern.

Problem 3. Consider the following fractal pattern, which one can use to create a fern. Use the pattern to produce the second (next) iteration of the fern:



How many line segments will occur in the n^{th} iteration?

Problem 4. You wish to find out how many NKU students have the dreaded disorder known as "SWARP". It's quite embarrassing, of course, and so you must be sensitive in collecting data on it. You use the single toss method to collect some data.

1. Explain how the method works.

2. You collect the following data on 110 students, randomly chosen from the student directory, using the single coin method: Estimate the true rate of SWARP.

Yes	87
No	23

Problem 5. In a certain class students either "get it" or they don't: this leads to a rather strange distribution of grades. Imagine that in a class of 30 students, 14 fail (and fail miserably, getting nearly zeros), and 16 get As (nearly 100%).

- Draw a histogram that represents approximately what the data would look like.

- What would the mean be, approximately?

- What would the median be, approximately?

Problem 6. As you no doubt recall, the formula for compound interest is

$$A = P(1 + r/n)^{nt}$$

where A is the amount (owed or in the bank, depending on whether it's a loan or a savings account), P is the initial principal, r is the nominal interest rate, n is the number of compoundings, and t is the number of years.

1. If you deposit \$1000 at 6% interest compounded semi-annually, how much would you have at the end of a year?

2. at the end of two years?

3. If you borrow \$1000, and pay a 3% charge up front, then pay 20% simple interest on all that at the end of a year, What is the effective APR, or Annual Percentage Rate?

Problem 7.

1. What is unusual about the Möbius band? What are its properties, and uses?

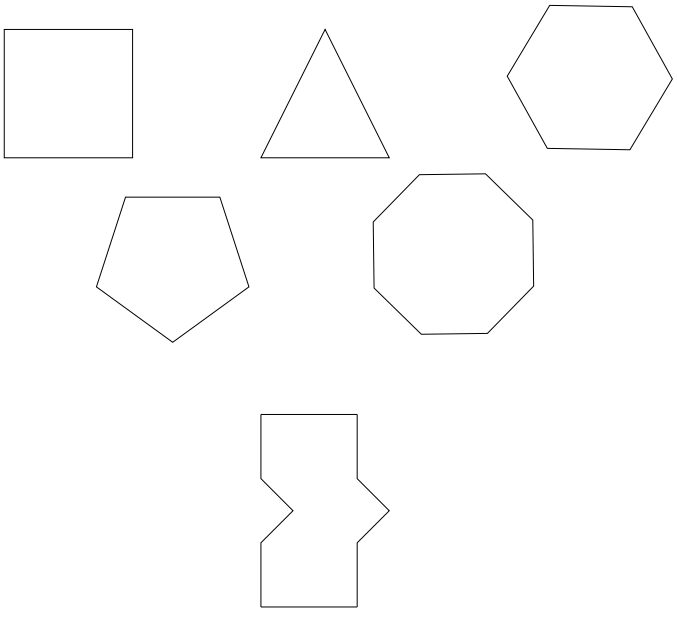
2. Which of the following images is a Möbius band? **Explain.**

Problem 8. Suppose that Fibonacci Nim is to be played starting with 29 sticks, by two wily characters, Sue and Bob, each of whom knows the winning strategy.

1. If Sue goes first, and Bob goes second, who is guaranteed a win if each plays well? (Explain!)

2. Write a plausible sequence of the first six plays (three each), assuming that each player is trying to win by whatever means possible. That is, each player should be making smart moves. Justify your moves.

Problem 9. Which of the following shapes can be used to tile the entire plane, using that tile **exclusively**? [For those which cannot be used, put an "x" in the middle, and explain why not.]



Problem 10.

1. Define the golden rectangle.

2. What are its amazing properties?

3. Demonstrate how the Fibonacci numbers can be used to approximate the golden rectangle.

Problem 11. As the desk clerk at the Motel ∞ (where there's a room for every natural number, and a natural number for every room), you encounter the following situations while the motel is "full": how do you handle each? (You can only ask each guest to move once, you may not put multiple clients in a room, and there will be no new construction or sleeping on the sofas!)

1. A pregnant woman and her husband show up.

2. A dozen unrelated strangers show up.

3. A bus with natural number labelled seats shows up, full.

4. Three busses with natural number labelled seats shows up, each full.