Fractals: A Brief Overview

Harlan J. Brothers, Director of Technology
The Country School   Madison, CT

This presentation provides a broad and basic introduction to the subject of fractal geometry.

My thanks to Michael Frame at Yale University for the use of many of the photos and graphics that appear here. His fascinating and comprehensive treatment of the subject can be found at:

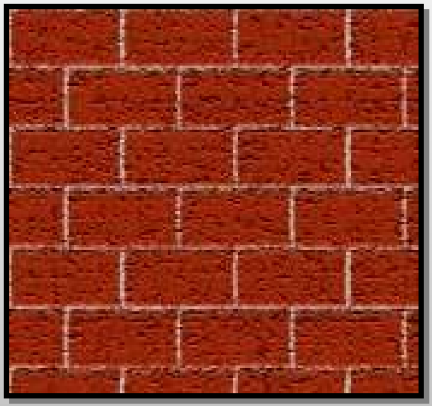
- http://classes.yale.edu/Fractals/ .

Familiar Symmetries

We commonly recognize when shapes demonstrate symmetry under the three familiar transformations of reflection, rotation, and translation.

Reflection

Rotation

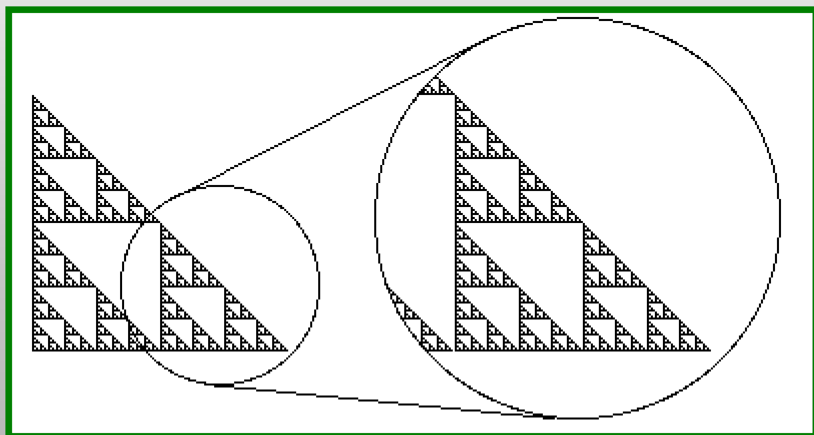


Translation

Scaling Symmetry

Fractals demonstrate a fourth type of symmetry; they possess "self-similarity."

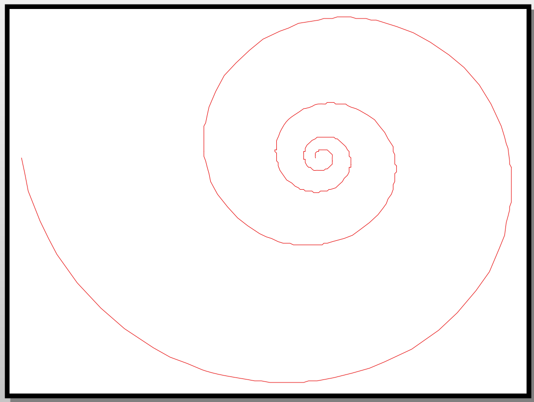
Self-similar objects appear the same under magnification. They are, in some fashion, composed of smaller copies of themselves. This characteristic is often referred to as "scaling symmetry" or "scale invariance."



Sierpinski Gasket

Scaling Symmetry

Not all self-similarity, however, is of a fractal nature. Objects like spirals and nested dolls that are self-similar around a single point are NOT fractal.



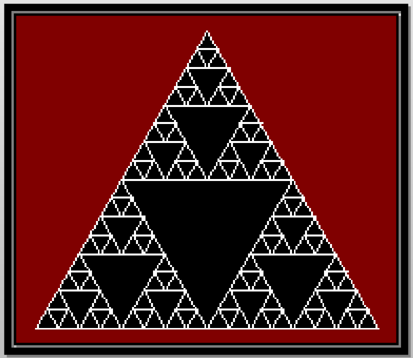
Not fractal

Not fractal

Fractals

In the broadest sense, fractals can be divided into two categories:

- objects that occur in Nature, and
- mathematical constructions.



Fractals in Nature

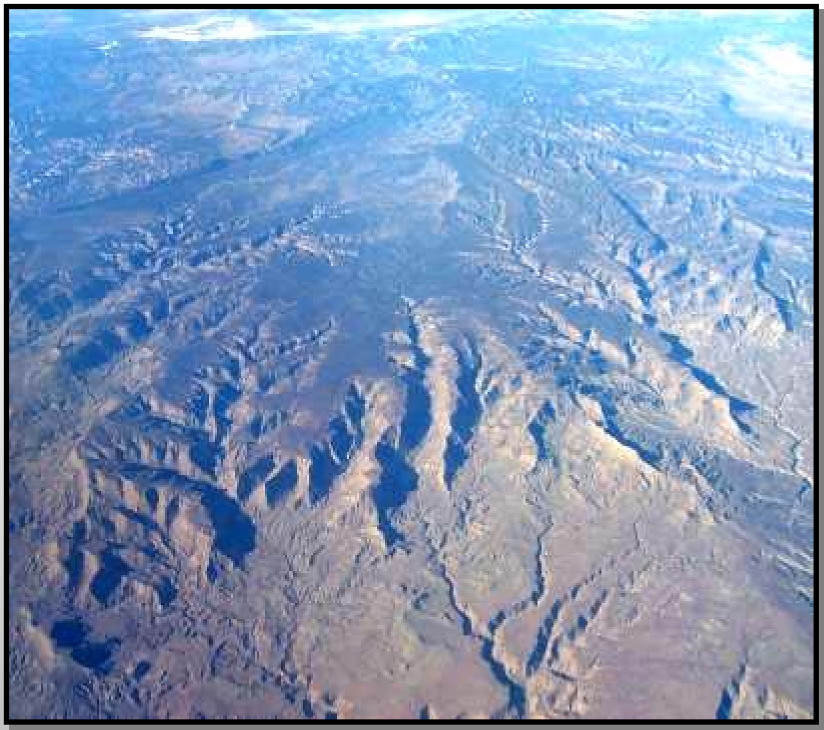
Natural objects exhibit scaling symmetry, but only over a limited range of scales. They also tend to be “roughly” self-similar, appearing more or less the same at different scales of measurement. Sometimes this means that they are statistically self-similar; that is to say, they have a distribution of elements that is similar under magnification.

Fractals in Nature

Trees

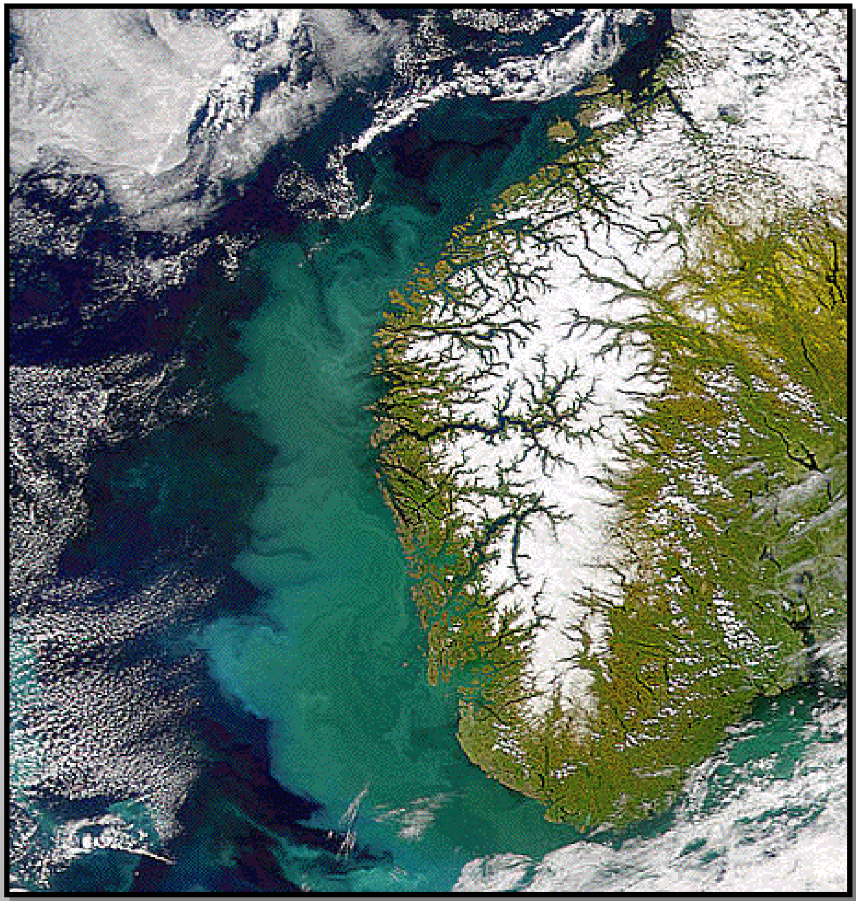
Ferns

Fractals in Nature

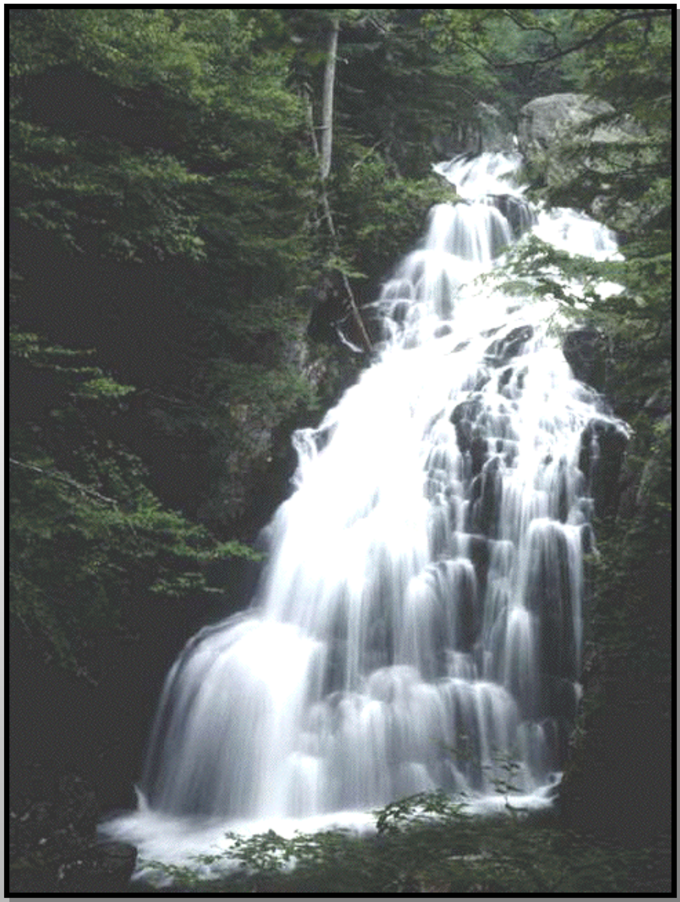


Mountains

Fractals in Nature



Coastline and snow fields of Norway

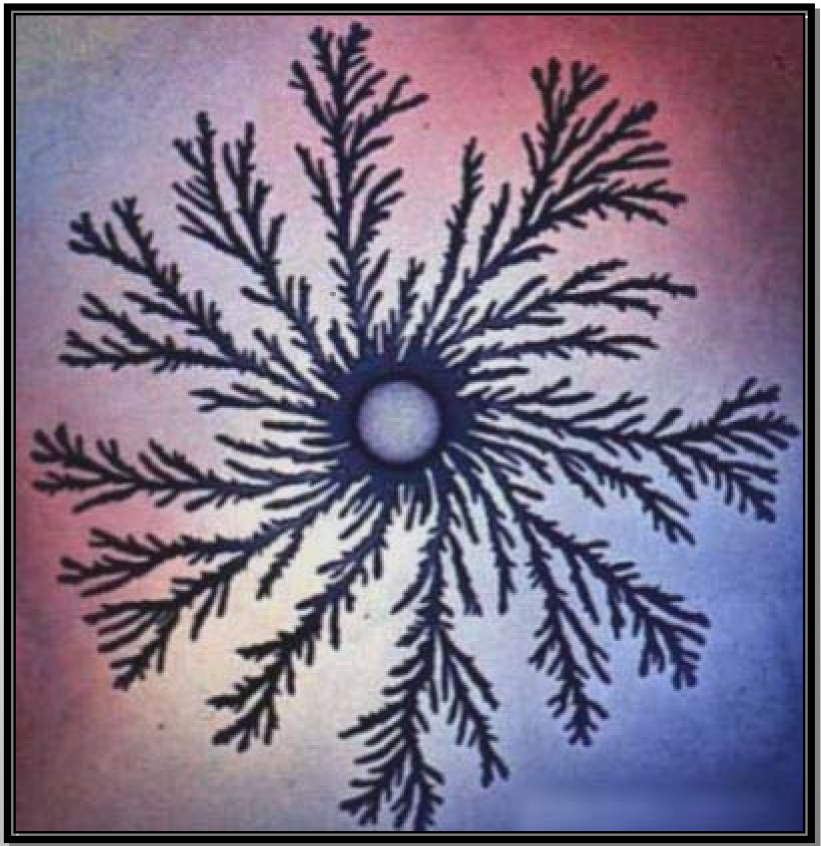


Waterfall

Fractals in Nature

Clouds

Fractals in Nature

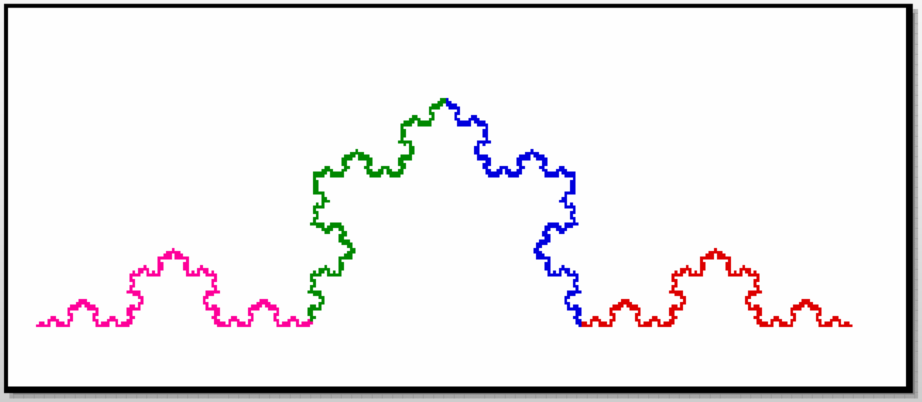


Bacterial colony
(courtesy E. Ben-Jacob)

Lightening

Mathematical Constructions

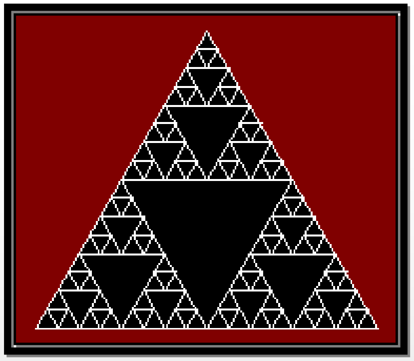
In contrast to naturally occurring fractals, mathematical fractals can possess an infinite range of scaling symmetry. The more common constructions also tend to be exactly self-similar.



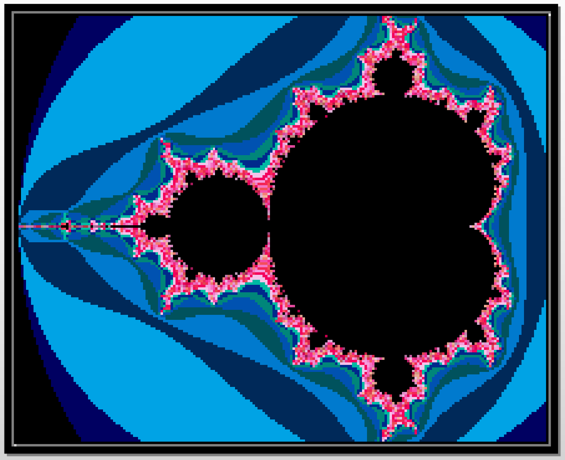
Koch Curve

The Koch curve above is composed of exactly four copies of itself. Can you construct it from just two?

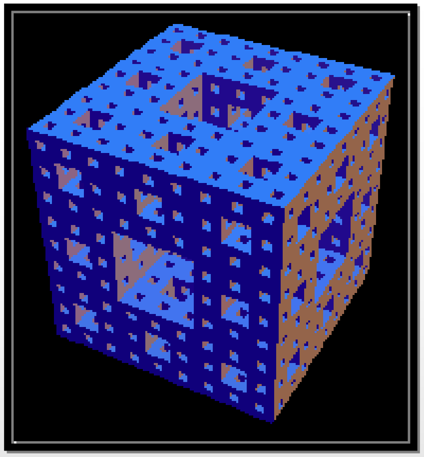
Mathematical Examples



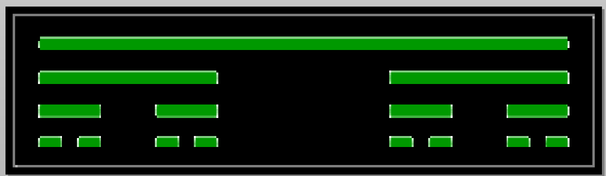
Sierpinski Gasket



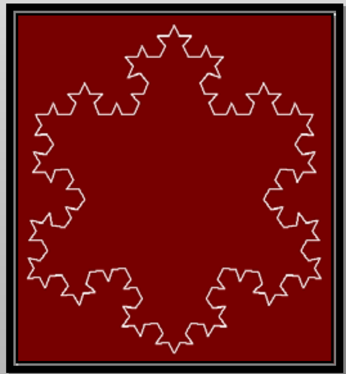
Mandelbrot Set



Menger Sponge



Cantor Comb



Koch Snowflake

Scale Invariance

The fact that a fractal object is, in some sense, composed of smaller copies of itself, has interesting implications. One of these is that when we examine a fractal shape without a suitable frame of reference, it is often impossible to tell the scale of magnification at which it is being viewed.

For natural phenomena, this translates to uncertainty with respect to the distance, extent, or size of the object. We end with two examples of scale invariance.

What do the following two images look like to you?

Image 1

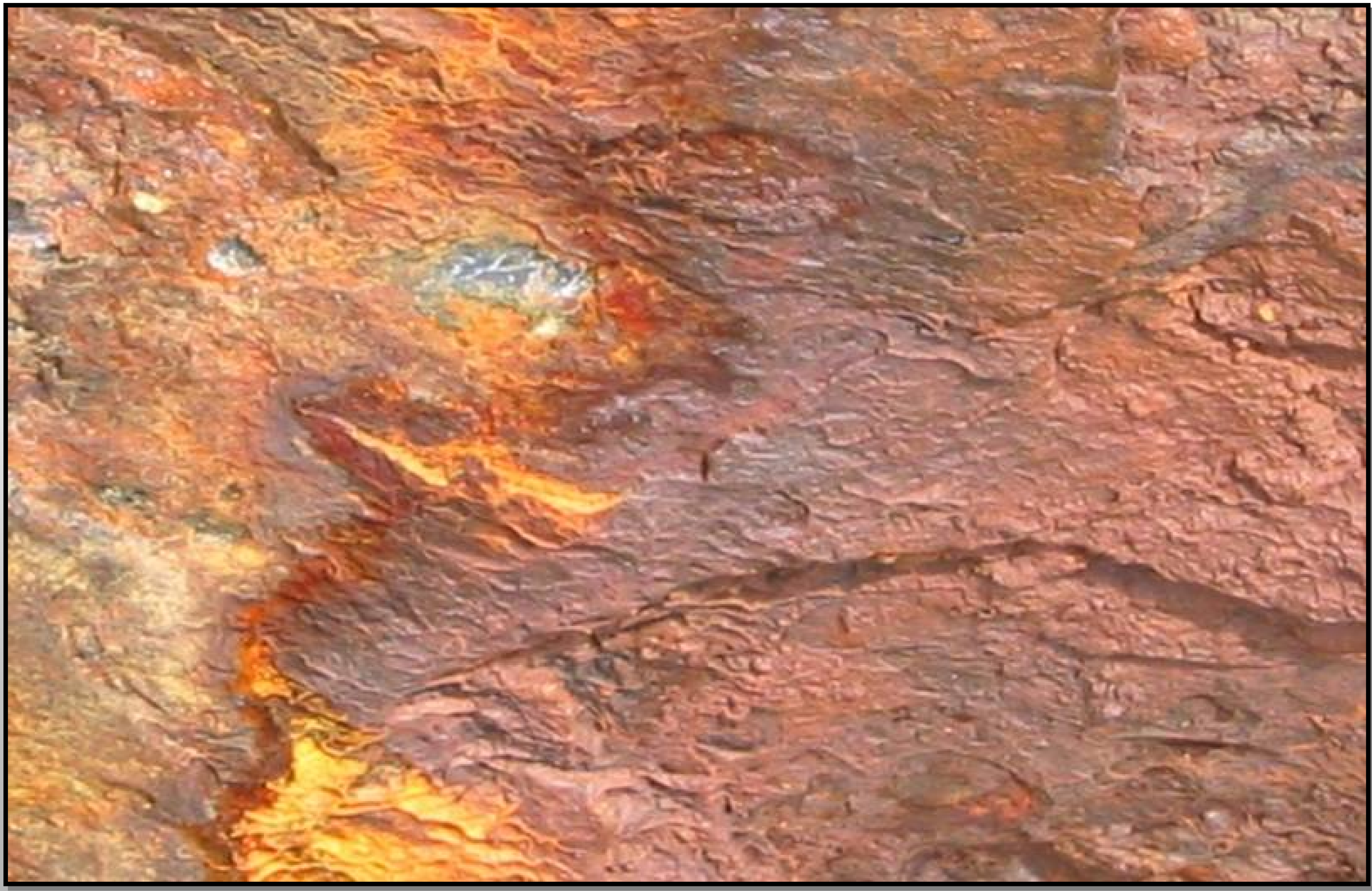


Image 2

Image 1 (another view)

Image 2 (another view)

Discussion

Depending on who you ask, the preceding images may look like satellite or aerial photos, rock formations, or photomicrographs.

This simply illustrates the fact that certain natural processes, like erosion or the formation ice crystals, follow patterns that can be repeated at many scales of measurement. Without a frame of reference, a photograph of a rock sitting one meter away can effectively look the same as a boulder several meters away or a cliff hundreds of meters distant.

With a knowledgeable eye, one sees a natural world that abounds in fractal shapes.

Author Information

Harlan J. Brothers
Director of Technology
The Country School
Madison, CT 06443
Tel. (203) 421-3113
E-mail: harlan@thecountryschool.org