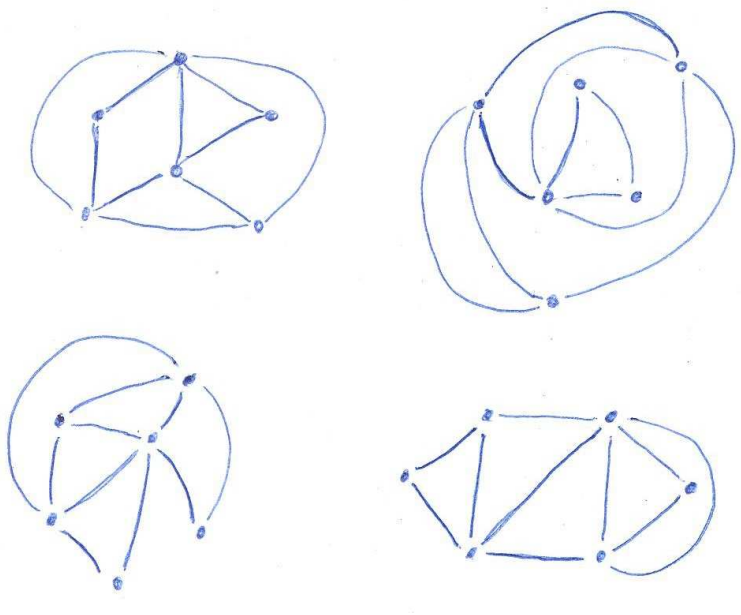
MAT385 Test 3 (Spring 2009): Graphs, Trees, and algorithms

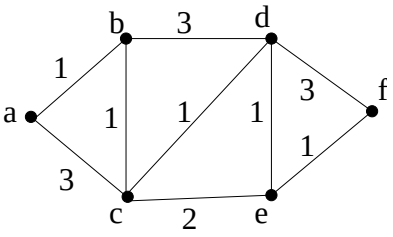
Name:

Directions: Points of each problem are marked. Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1. (10 pts) Two of the following graphs are isomorphic; the others are different. Find the isomorphic pair, and explain how you know.



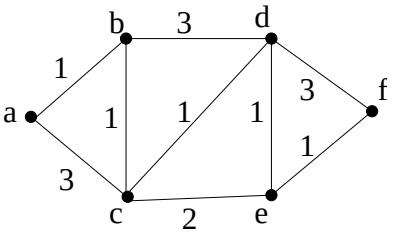
Problem 2. (10 pts) Consider the following graph:



a. Use Dijkstra's algorithm to find the shortest path from node a to node d. Give the path, and minimal length.

b. Perform one iteration of the Bellman-Ford algorithm to find the shortest paths from d to every other node **using paths of length 2 or less**.

Problem 3. (15 pts) Same graph as in Problem 2:



a. (3pts) Write the adjacency matrix for the graph (use the weights for the matrix entries). Is this an efficient storage method?

b. (2pts) Use an adjacency list to represent this matrix.

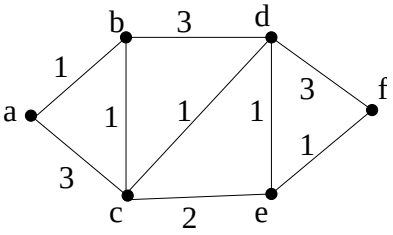
c. (2pts) Use any algorithm (indicate which you're using) to create a minimal spanning tree for the graph. Is the tree unique?

d. (2pts) How many nodes may serve as starting points for Euler paths (draw a sample route from each starting node)?

e. (4pts) Determine how many Hamiltonian circuits exist for the graph that start from node a.

f. (2pts) What is a planar graph? Verify Euler's formula for this graph.

Problem 4. (10 pts) Same graph: use both a depth-first algorithm and a breadth-first algorithm to write the nodes of the graph:



a. Depth-first:

i. Starting from a:

ii. Starting from e:

b. Breadth-first:

i. Starting from a:

ii. Starting from e:

Problem 5. (10 pts) Consider the algebraic formula

$$x * (3 - (2 \div y)) + \cos(5 * y + \pi)$$

a. (4pts) Draw the expression tree for this formula.

b. (2pts) Perform a preorder traversal of the tree.

c. (2pts) Perform a postorder traversal of the tree.

d. (2pts) Consider an algebraic formula AE containing only binary operations and variables. What is the maximum number of binary operations that could appear in AE if its expression tree has depth d?

Problem 6. (10 pts) Given the following list of students: ashworth, cooper, daniels, dringenburg, gieske, green, landis, mcgee, neilan, povkov, scudder, tobin, yannarella. (You may abbreviate the names to their smallest unique lengths.)

a. (2pts) What is the minimal depth of the binary tree search possible for this data? What is the maximal depth?

b. (3pts) Create an example of the most balanced binary search tree possible for this data (spread out!).

c. (3pts) Here is a random order of those names: scudder yannarella ashworth cooper neilan tobin mcgee gieske daniels povkov dringenburg green landis.

   Create the corresponding binary search tree for this ordering (turn the paper sideways!).

d. (2pts) Any algorithm that solves the searching problem for a 13-element list by comparing the target element x to the list items must do how many comparisons in the worst case? (Now, how did you do in parts a and b?)