MAT115 Final (Fall 2010)

Name:

Directions: Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **You must skip one problem from 1-8 (but not 9-11). Write "skip" on the problem. Good luck!**

Problem 1: (10 pts)

A farmer (Joe) is taking his brother (B), his girlfriend (G), and his bag of Dove chocolates (D) home from the market, when he comes to a river, with a two-seater boat on the shore. He's got to get everyone and everything in his care across the river, but the boat can only take two of the people (or things) at a time in the boat. If he leaves his girlfriend with his brother, his brother will hit on her; if he leaves his girlfriend with his Dove bars, he won't have any left when he gets back. Any person can pilot the boat.

Show how Joe manages things so as to get everyone and everything safely to the other side.
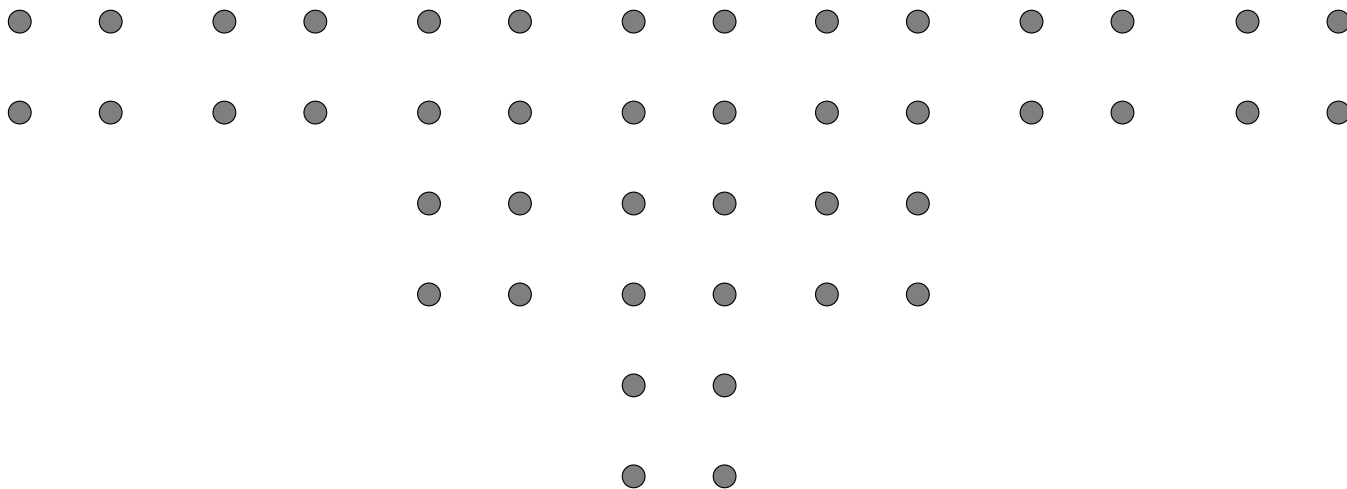
Problem 2: (10 pts)

a. Use Egyptian Multiplication to multiply 23*51.

b. Write 1024_{10} in base 5.

c. Write 1024_5 in base 10.

Problem 3: (10 pts)

a. Draw all simple graphs with exactly four vertices (there are exactly 11). Show which graphs are dual to which other graphs.

b. Draw either a figure-eight knot, or either five knot.

Problem 4: (10 pts) Fibonacci numbers

Part I: Suppose 6 and 9 were consecutive Fibonacci numbers; what would be

a. the Fibonacci following 9?

b. the Fibonacci preceding 6?

Part II: You're playing the game of Fibonacci nim:

a. What do you do if you are given the choice of going first or second given the starting value of 55 sticks?

b. Are you in a winning position if it is your turn, your opponent just took 3 sticks, and you are staring at 36 sticks? What do you do next?

Problem 5: (10 pts) Geometry

    a. Which of the following rectangles is the closest to golden?
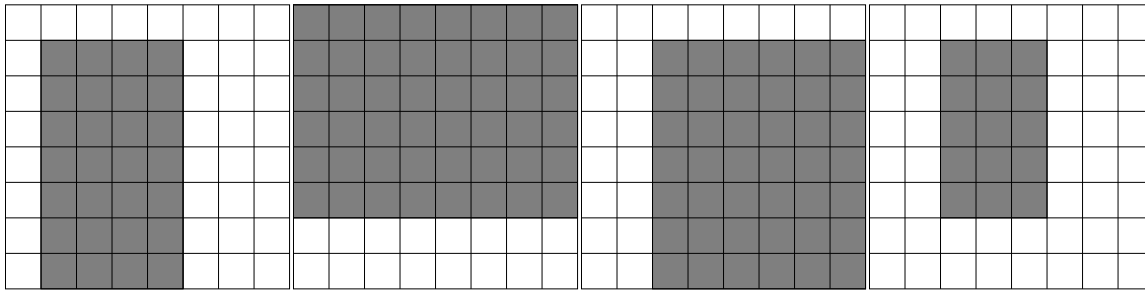
    b. Given a golden rectangle: what do you get when you remove the largest square possible from the rectangle?

    c. Examine the figure below:

i. The edge in this object crosses itself. What kind of knot results if you trace the edge?

ii. What is this object called, and what happens if you cut it down the middle?

Problem 6: (10 pts) Fractals

The following figure shows the initial line segment, and the line segment after application of a simple process. One could say that the initial stick has been broken into four equal sized line segments:

a. Perform the next iteration to the figure on the right.

b. Draw either the Sierpinski Triangle, or the Koch Snowflake.

Problem 7: (10 pts) Elections

a. What is the difference between a plurality and a majority in an election?

b. (6 pts) Consider the following data, which represents six people voting on three candidates (A, B, or C), from "first to worst".

Order of Candidate	Bob	Bill	Bjorn	Brenda	Bobbie	Billie
First	A	B	A	A	B	C
Second	B	C	B	B	A	B
Third	C	A	C	C	C	A

i. Use plurality to determine the victor, if possible.

ii. Use Borda Count to determine the victor, if possible.

iii. Use "vote for two" to determine the victor, if possible.

Problem 8: (10 pts) More Geometry

    a. Draw an octahedron, and its dual.

    b. Draw Borromean rings.

Problem 9: (10 pts) Dynamics: Given the logistic map

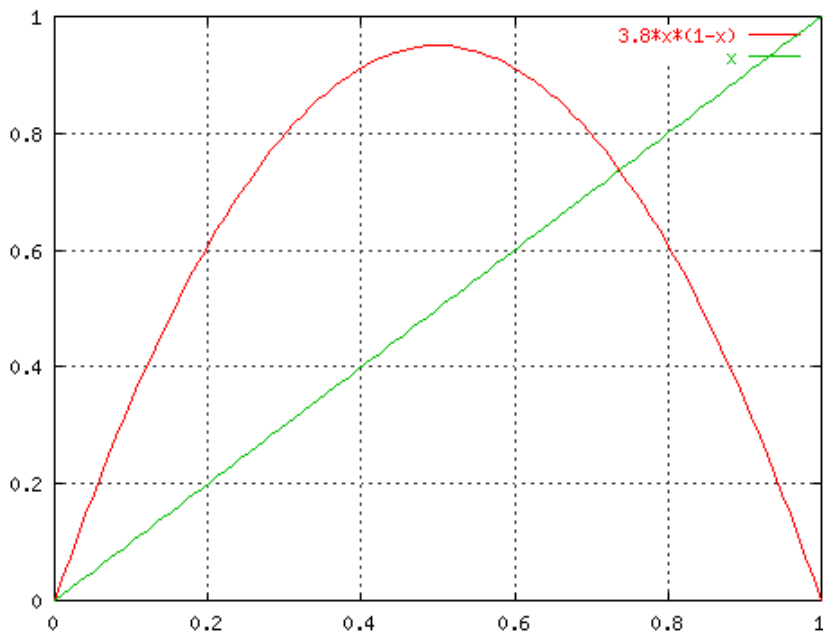$$x_{n+1} = rx_n(1 - x_n)$$

with $r = 3.8$ and $x_0 = .2$.

a. (3 pts) Compute the first three iterates x_1, x_2, and x_3.
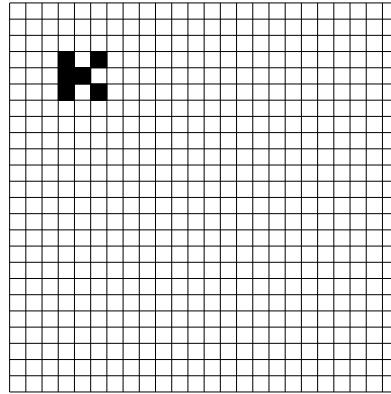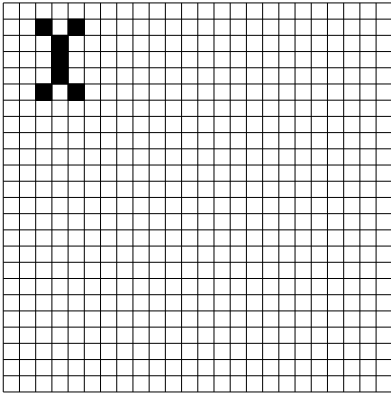
    i. $x_1 =$

    ii. $x_2 =$

    iii. $x_3 =$

b. (5 pts) Use the following parabolic plot to carefully cobweb the first three iterates:


c. (2 pts) What is our informal definition of chaos?

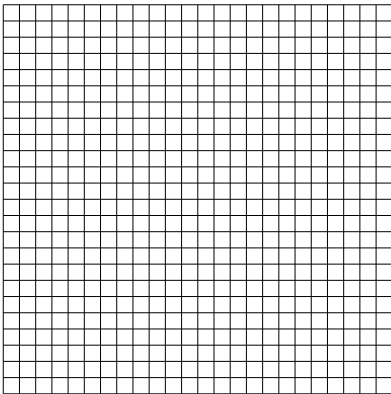**Problem 10:** (10 pts)

a. Given the following initial populations in the game of life, draw the next iteration in the space adjacent to each (if you make a mistake, just cross it out and try again):
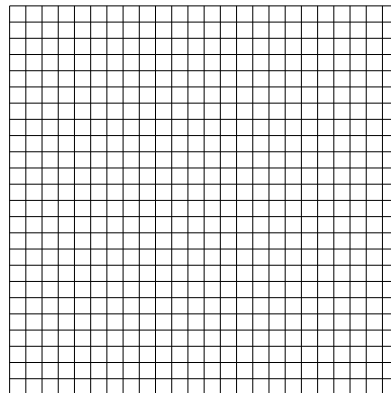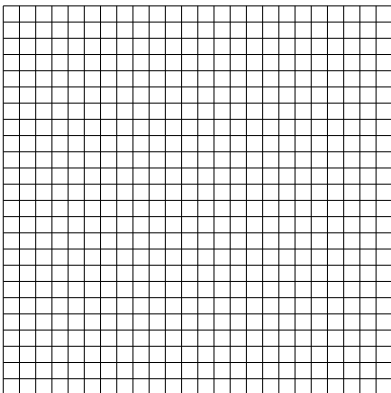
b. Give examples of the following patterns in the Game of Life:

i. On the left grid, a pattern that never changes; on the right grid, a pattern that oscillates.
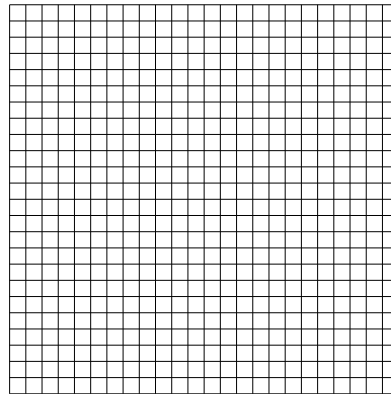
ii. On the left grid, a pattern that goes extinct; on the right grid, a pattern that will fly.

Problem 11:   (10 pts) The Game of Logic: for which of these trios is the conclusion correct?
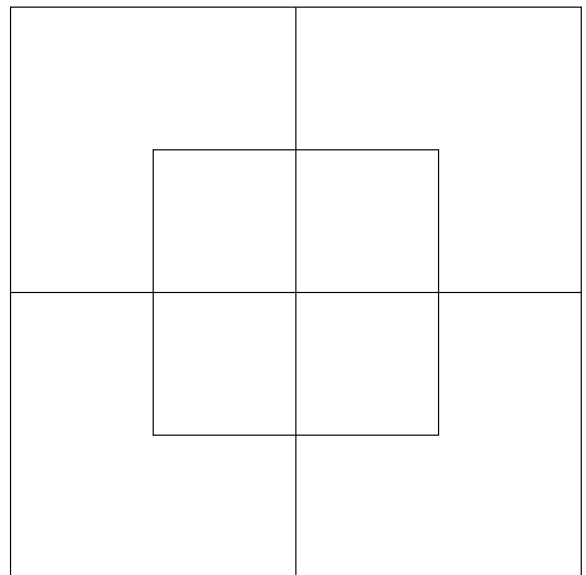
Demonstrate the use of the trilateral diagrams of Lewis Carroll to help you decide....

a. No birds, except peacocks, are proud of their tails;
Some birds, that are proud of their tails, cannot sing.
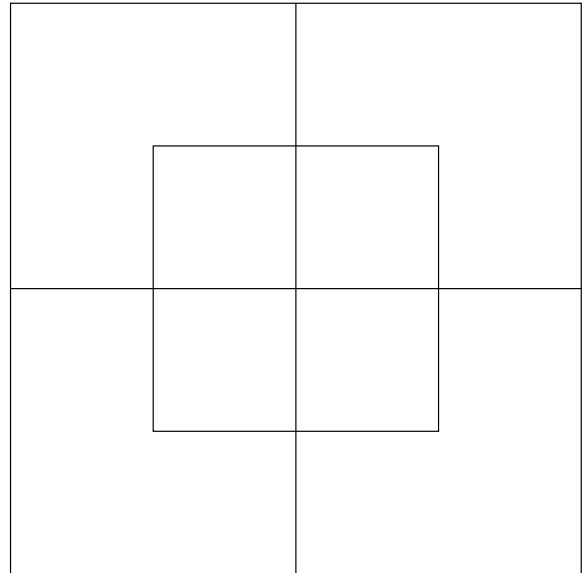
Some peacocks cannot sing.

b. No emperors are dentists;
All dentists are dreaded by children.

No emperors are dreaded by children.