THEOREM 1 The Fundamental Theorem of Calculus, Part I Assume that $f(x)$ is continuous on $[a, b]$ and let $F(x)$ be an antiderivative of $f(x)$ on $[a, b]$. Then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Figure 1: The Fundamental Theorem of Calculus, Part I (from Rogawski)

Lab for the Fundamental Theorem of Calculus

So far, we've got some great ideas for how to approximate the area under a curve (which work especially well for real data, which tends to be just a discontinuous set of points).

But what if we actually have a formula for a function $f(x)$, and its graph is some nice, continuous curve? Can we do better than approximate the answer with a Riemann sum? We can (and it's easy as pie, too!), provided we know one thing: an anti-derivative of f.

An **anti-derivative** of f is a function F whose derivative is f: $\frac{dF}{dx} = f(x)$.

For example, consider the integral $\int_a^b x dx$.

1. Draw a graph of the function ($f(x) = x$) on the interval $[a, b]$ (assume $0 < a < b$). Interpret the integral on the graph.

2. The rectangle rules R_1 and L_1 should average to give the correct result (since it's a trapezoid, and the average of these two rules is the trapezoidal method!). Compute R_1 and L_1, and average them.

3. Now use the Fundamental Theorem (you need to find an anti-derivative) and compare your answers.

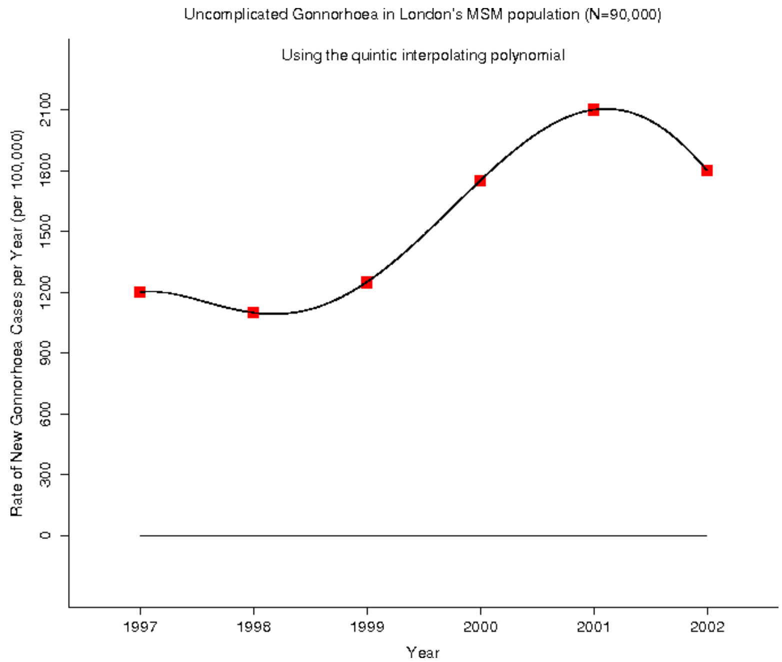

Figure 2: Uncomplicated gonnorhoea in London's MSM population (estimated at 90,000)

Now last time we talked about this gonnorhoea data, and Arthur didn't like the function that the authors used to interpolate (connect) the dots (they used straight line segments). He didn't like the lack of smoothness. We agreed that we should be able to find a 5^{th} degree polynomial (a **quintic**) that fit the data. You can see it in the graph above. Here is the function:

$$q(x) = 1200.0+78.33(x-1997)-450(x-1997)^2+341.67(x-1997)^3-75(x-1997)^4+5(x-1997)^5$$

Now: try to evaluate

$$\int_{1997}^{2002} q(x)dx$$

1. by hand, and
2. using your calculator.

Interpret the results, and compare to the approximations to the integral you got in the last lab, using Riemann sums.