Section 5.5: Net or Total Change as the Integral of a Rate

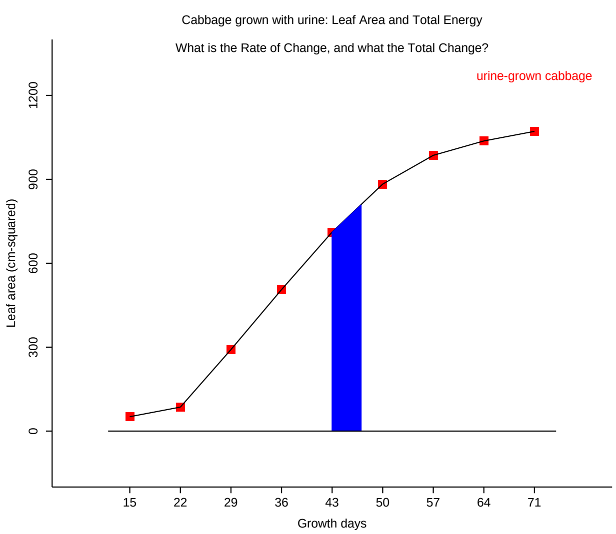
Big picture: Calculus is often about measuring change. If we have a rate of change, we may be interested in calculating the total change that occurs over the inteval on which the change is occurring.

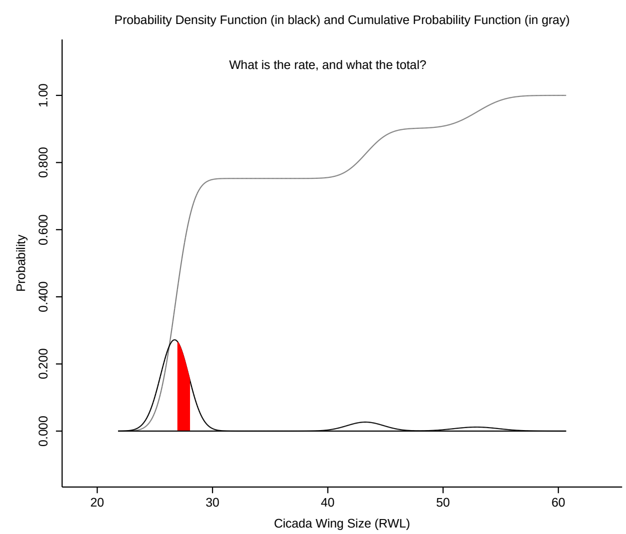
Our author suggests that we now consider integrals as representing changes, rather than areas. He gives us another formula for the integral, in general:

$$\int_{t_1}^{t_2} s'(t)dt = s(t_2) - s(t_1)$$

We think of the function in the integrand as a derivative, or **rate of change** of the function s. The FTC says that this is an appropriate way to think about the two sides.

In two of the problems we've already considered, the urine and the cicadas,



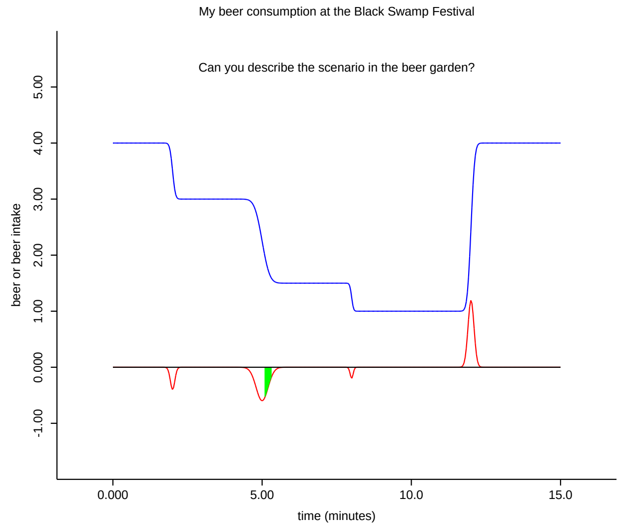


we had something changing: the change was in time for the leaf size for cabbages, and the change was in size of the right wing length for the population of cicadas. We related the leaf size to the energy that the leaf was capable of contributing to the plant.

Over the weekend I was at the Black Swamp Arts Festival, in Bowling Green, Ohio, and a couple of guys I was drinking beer with asked me "What's calculus, anyway?" I think that they quickly suspected that that was a mistake (especially while drinking beer), but the example I gave them involved beer. The example I gave them was a man, drinking a beer, who watches the amount of beer in his glass change over time. There are two things impacting this: the rate at which he drinks his beer, and the rate at which it gets refilled.

Problems: In the graphs of this lab,

1. Identify the rates of change.
2. Identify the total changes that are occurring.
3. Explain the meaning of the small areas highlighted under one of the graphs in each of the three plots.



4. For those plots with two graphs, explain and justify the relationship between the two graphs.

5. For the plot with only a single graph, draw a plausible second graph related to the concept "Net or Total Change as the Integral of a Rate".

Another observation to make is that, using Leibniz's notation,

$$\int_{t_1}^{t_2} s'(t)dt = \int_{t_1}^{t_2} \frac{ds(t)}{dt} dt = \int_{t_1}^{t_2} ds(t) = s(t_2) - s(t_1)$$

We're adding up infinitesimal bits of s, $ds(t)$, as we march from $t = t_1$ to $t = t_2$. The discrete analog of this is that we're adding little chunks (via Riemann sums) to get approximations:

$$s(t_2) - s(t_1) \approx \sum_{i=1}^{N} \Delta s(t_i) = \sum_{i=1}^{N} s'(t_i)\Delta t_i$$